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Strategies for Introducing Pitch-Class Set Theory in the Undergraduate Classroom

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Even as pitch-class set theory has been widely integrated into undergraduate music theory curricula, it has remained one of the more difficult topics of the typical undergraduate program for instructors to illuminate and for students to master. A number of factors contribute to this phenomenon: the atonal repertoire for which it was devised is shunned by many students; the overt affinity with mathematics in many presentations of the theory makes it anathema to some; the theory's particular employment of inversional equivalence has no clear antecedent in musical theories already familiar to undergraduates; and the theory's jargon seems mysterious and dense to the uninitiated. As Joseph Straus once put it, "Atonal set theory has a bad reputation."[1]

My purpose here is to offer practical suggestions regarding the initial presentation of the theory to help students quickly acquire an easy fluency with both its fundamental tenets and its analytic value. The teaching ideas offered here, depending on the instructor's own curricular priorities, may serve as a springboard to more advanced exploration of related theoretical concepts or to beginning analyses of post-tonal music. In either case, to be successful students must master the conceptual framework of the theory and develop some fluency in its fundamentals. These are my goals as I introduce this powerful analytic tool to my own students.

The Pedagogical Obstacles

During an office hour many years ago, an above-average student in my undergraduate theory course asked me what topics remained for us to cover in that semester. When I mentioned pitch-class set theory, she asked with great trepidation, "Is that the atonal

Many of the ideas in this essay were first presented in a poster session at the 2007 conference of Music Theory Midwest at the University of Kansas. I am indebted to the participants in that session as well as Tim Best, Melissa Hoag, and the anonymous reviewers of this article for their help in articulating this pedagogical approach and its rationale.

[1] Joseph N. Straus, "A Primer for Atonal Set Theory," *College Music Symposium* 31 (1991): 1.

music with all the math in it?" This response represented several overlapping apprehensions she had inherited from upperclassmen who had already completed the course. Of course, the repertoire to which this theory is most often applied is no more or less intrinsically "mathematical" than tonal music. As I learned, this student was attempting to articulate two fears: that the music we were about to study would sound like cacophonous nonsense, and that to understand it she would need to be a mathematician. While it is impossible to generalize about the perspectives of every student beginning a unit on set-class theory, a great many have one or both of these misconceptions.

Prior to beginning our post-tonal analyses early in their fourth semester of the theory curriculum, a large number of my music students know little more about the Second Viennese School than perhaps Schoenberg's name. Most of these students have never listened closely to an entire atonal work or studied such music in any detail. Even those students who have had some exposure to atonality or non-triadic music testify that they find it baffling, frustrating, and unattractive. It is no small wonder that, when they learn to what repertoire we are about to turn, many immediately become less interested, less receptive, more skeptical, and—as a result—more unlikely to engage meaningfully with the very theory that could "open up" this music for them.

If the impending focus on atonal repertoire is disconcerting to some students, the perception that our analysis will somehow translate the music into a mathematical equation is absolutely frightening. Straus puts it well:

> …set theory has had an air of the secret society about it, with admission granted only to those who possess the magic password, a forbidding technical vocabulary bristling with expressions like "6-Z44" and "interval vector." It has thus often appeared to the uninitiated as the sterile application of arcane, mathematical concepts to inaudible and uninteresting musical relationships. This situation has created understandable frustration among musicians, and the frustration has grown as discussions of twentieth-century music in the professional theoretical literature have come to be expressed almost entirely in this unfamiliar language.[2]

[2] Ibid., 1.

The explicit use of a science (mathematics) to explain an art form (music) is troubling and even offensive to some students. The abstraction required to turn pitch classes into numbers may strike some as illogical or at least counterintuitive. As David Mancini previously observed in this journal, "[Students] may have difficulty thinking about musical relationships in terms of integers, the traditional symbols of set theory. A particular stumbling block for some students is modulo 12 arithmetic…. A sensitive instructor should be aware of not only these specific difficulties, but of the frustration emanating from them as well."[3] (Straus's own textbook all but apologizes for asking students to "add" two notes together.[4]) Additionally, there are many music students for whom classes in math and other sciences, whether in their secondary or post-secondary education, were difficult and fraught experiences. For these undergraduates the artistry and instinctive aspects of music-making have always matched their aptitudes better than those needed for success in science-related fields, and mark the very reasons they are in degree programs training them to become opera singers or band directors rather than structural engineers or systems analysts. The prospect of mathematics impinging upon their musical studies is thus established as an obstacle in their minds even before pitch-class set theory is introduced in the classroom.

To mitigate these difficulties, I have developed the approach to introducing set theory described below. It addresses the problem of atonal music's "inaccessibility" by creating intimate familiarity with a brief—but complete—movement by Webern, and draws students into making their own analytic observations, first by asking the same questions we might ask about a tonal work, then by challenging them to identify and label the small groups of intervals (the set types) that govern the music's structure. The introduction of the theory itself delays mathematical abstractions such as integer representation of pitch classes and the ubiquitous clock-face metaphor for pitch-class space. Instead, I employ standard musical notation and a collection of playful, memorable metaphors to dissolve the "air of the secret society" that Straus describes. In certain respects, some methods provided here for explaining fundamentals of set theory are hardly revolutionary. Experienced instructors will recognize aspects of

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[3] David Mancini, "Teaching Set Theory in the Undergraduate Core Curriculum," *Journal of Music Theory Pedagogy* 5, no. 1 (spring 1991): 96.

[4] Straus, *Introduction to Post-Tonal Theory*, 3d ed. (Upper Saddle River, NJ: Pearson Prentice Hall, 2005): 48–49.

the given procedures for determining normal order, labeling set types, and so on. I do not believe a total overhaul of this subject's pedagogy is required; moreover, there is a finite number of good ways to explain each fundamental concept of set theory. Instead, the motivations for this essay are to suggest a carefully charted order in which to introduce the interrelated concepts of pitch-class set theory for the first time; to provide a detailed strategy for orienting students to the musical significance of each element of the theory; to exposit a small body of metaphorical images that students and instructor can draw upon together, facilitating easy discussion of the theory and its implementation; and (in sum) to provide a conceptual gateway to the theory that is neutral enough to dovetail with a wide variety of subsequent readings, musical analyses, and presentations of the theory's other features.

Before describing the teaching approach in detail, allow me to make clear that I find integer representation and the clock face appropriate tools in set theory and its pedagogy. My explicit avoidance of these common metaphors should not be read as an implicit objection to their use. Indeed, the comparisons below of sets to buildings and to M&Ms are no less abstractions than is a mod-12 system in which C = 0. I have found, however, that early avoidance of specifically mathematical abstractions, coupled with consistent representation of sets in musical notation, helps to break down the barriers described above. By building facility in finding prime forms and performing basic transformations using *at first* only musical notation, many students develop greater cognition of the theory's operations as they manifest in pitch space (and often do so with less anxiety). One goal of this introduction to the theory is to prepare students for detailed analytic work using set classes, which certainly may involve abstraction to mathematical representations that—thanks to this foundation—will more clearly represent musical entities in the students' minds.

Cocoa Puffs and the Need for the Theory

Whether this theory's reputation is deserved or not, the apparatus required to apply it fluently is substantial. While some of its components have analogs in tonal theory (transposition and modulation might be viewed as a precursor to T_n, for instance), set theory's applications and vocabulary differ considerably from analytic methods with which students are already acquainted.

The result is a steep learning curve. As students wrestle with the conceptual hurdles, a worthwhile question emerges: "Why are we doing this?" For all the (perceived) difficulty in learning this theory, what will it really tell us about a given musical work?

It is helpful to students to address these questions of musical relevancy even before they typically think to ask them. Instead of describing the theory in the abstract "from scratch," beginning the inquiry via investigation of a carefully selected work can demonstrate the analytic need for a method of classifying groups of notes according to their interval content. To that end, I begin with the third of Webern's *Fünf Sätze für Streichquartett*, Op. 5.[5] Figure 1 shows the opening of this movement.

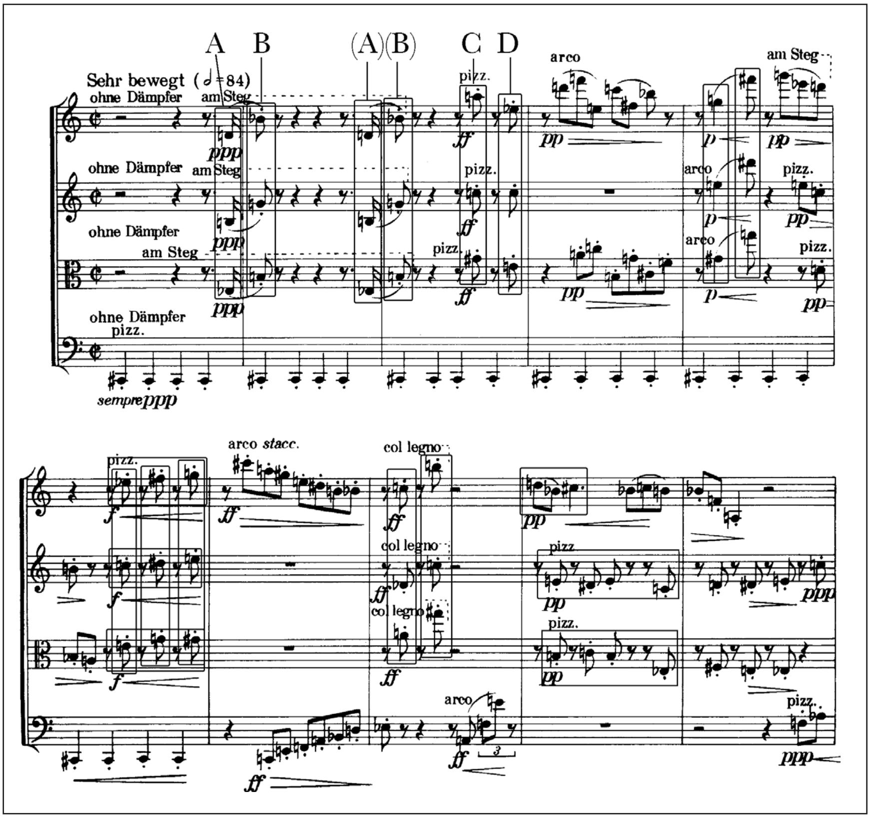

Figure 1 - Webern, *Fünf Sätze für Streichquartett*, Op. 5, No. 3, mm. 1-10; instances of (014) marked.

[5] Miguel Roig-Francolí offers a detailed analysis of this movement in chapters 3 and 4 of *Understanding Post-Tonal Music* (New York: McGraw-Hill, 2008) to which the in-class work I describe here might serve as an introduction.

As preparation for class, students listen to the entire movement several times while following along in a score. The in-class discussion opens with broad questions that I've asked students to consider in advance:

• Do you think C# has a special significance to this movement? Why or why not? If so, is its significance *audible*? Is it a pitch center or a "tonic?" How does it frame the movement?

• How would you describe this movement formally? How many sections are there? How are they delineated?

• Describe the texture at each moment of the work. Are any portions homophonic? heterophonic? monophonic? polyphonic? Are there any ostinati? canons?

• Are there any elements of repetition in this music? Is any musical element (chord, melody, rhythm, section) ever repeated or recapitulated? What might a listener "latch onto?"

These kinds of questions would be appropriate for any of the tonal repertoire students have already studied for several semesters, and thus serve to make students more comfortable and familiar with this movement. Even as we grapple with this music, though, it becomes clear that none of the analytic methods students emphasized in the music theory curriculum to this point is quite satisfactory to this music. C# is crucial to this movement, but not in the same sense as a traditional tonic, and while it is possible to speak of the work's large-scale organization by noting sectional divisions and motivic repetitions, its harmonic and melodic content resists analysis using students' current vocabulary. Aside from stating the obvious—the music is chromatic and atonal—the class realizes it lacks the theoretical language and tools to describe individual sonorities or melodic lines with any rigor.

As students express their first ideas about the work's organization in this discussion, I guide them to consider the harmonies that open the work above the cello's C# pedal point. Students can tell quickly that the upper strings' sonorities in mm. 1–3 are not tertian, but there is something consistent about their construction. After playing these chords in isolation at the keyboard and reducing them to a single staff on the chalkboard (see Figure 2), that commonality becomes clear. Each chord is made of three pitches that can be arranged as either a minor third stacked atop a half step or as a half step atop a minor third.

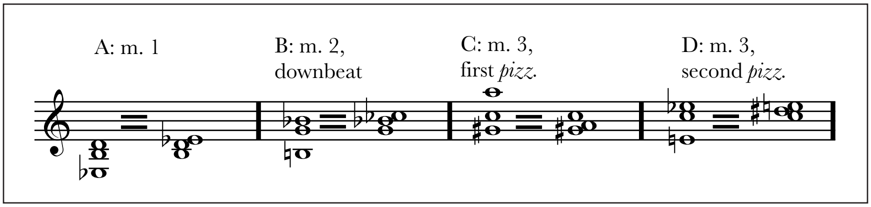

Figure 2 - Preliminary analysis of four sets from Webern, Op. 5, No. 3.

This small discovery helps me to define to my students the need for pitch-class set theory. Clearly Webern is working here with a certain "type" of chord in a consistent way. I encourage students to invent a temporary label for this chord type—an enthusiastic student once called it a "Cocoa Puff"—but we have no *meaningful* way to label it. Following Stefan Kostka and Miguel Roig-Francolí,[6] I demonstrate that the label *major triad* is analogous to the label that is lacking here. *Major triad* does not refer to any specific pitch or registral deployment, but rather a whole category of sonorities that share certain intervallic features among its members. Students readily recognize that the pitches D, F#, and A, however arranged in pitch space, constitute an instance of a major triad. C, E, and G together constitute another different major triad: even though these two entities do not share common pitches or (necessarily) similar registral arrangement, they both belong to that class of musical objects we call major triads.[7] This is the sort of label we need for Webern's chords—and this is the label that pitch-class set theory can provide. By explaining to students that set theory provides a single logical name for all four of the chords in Figure 2, the instructor orients students in advance to the power of the theory: providing a way in which groups of notes dissimilar in pitch content and register can be viewed as "equivalent" based upon their intervallic content.

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[6] Stefan Kostka, *Materials and Techniques of Twentieth-Century Music*, 3d ed. (Upper Saddle River, NJ: Pearson Prentice Hall, 2006), 185; Roig-Francolí, 74.

[7] In fact, set class names are more directly analogous to the category "major and minor triads," as major and minor triads are inversionally equivalent in this theory—but to avoid early confusion I choose not to introduce the issue of inversional equivalence to students at this stage.

Defining Types of Equivalence

Many domains of equivalency crucial to pitch-class set theory also pervade theories of tonal music. Octave equivalence and transpositional equivalence are transparent to most students because they are essential to understanding tonal repertoire. Thus, they have no trouble recognizing that multiple manifestations of {C, D, E} can be viewed as equivalent regardless of the registers in which any of its elements appear. {C, D, E} and {F#, G#, A#} have a contrasting obvious equivalence based upon their successive interval content. It is worthwhile to point out and define these equivalencies to students, but most intuitively negotiate them without difficulty. The ease with which a typical class will generate Figure 2 demonstrates this phenomenon.

The next step, then, is for students to describe with precision the kinds of "sameness" that led them to apply the same arbitrary label to all four chords ("sets") in Figure 2. One way to do this is put the question negatively: "What differences among these four sets do we have to ignore in order to say they're 'the same?'" Students' responses lead them to a catalog of equivalence types, most of which are familiar from tonal theory. For instance, when someone points out that the sets' registral arrangements are not all the same, a discussion of octave equivalence follows. I explicitly define each equivalence class as class members "discover" them:

• *Octave equivalence*. No matter in which octave any given note of the set appears, the set is still the same set. For instance, in Figure 2, the movement of Set A's E♭3 up an octave to E♭4 doesn't change the set's identity.

• *Enharmonic equivalence*. Put simply, the spelling of a note does not change its identity. Our work with Sets B and D in Figure 2 made use of this equivalence. (Some students, after weeks of respelling enharmonic diminished seventh chords to imply different keys, are relieved to learn that they may now use whatever enharmonic spellings they find convenient!)[8]

[8] A by-product of enharmonic equivalence is that, for the purposes of this theory, we may treat intervals as enharmonically equivalent as well. Throughout my presentation of this theory, I convert chromatic intervals to diatonic equivalents (e.g., augmented fifths become minor sixths) to facilitate easier comparison of interval sizes—a crucial component of classifying sets—as well as to remind students of this contrasting attitude toward enharmonicism in relation to tonal theories.

At this stage it becomes possible to define (or reintroduce[9]) the term *pitch class* for students. Assuming octave equivalence and enharmonic equivalence, every pitch that *sounds* like a C regardless of its register or spelling is a member of the same pitch class. (To test for comprehension, ask students how many different pitch classes exist in traditional equal temperament.)

• *Transpositional equivalence*. Our label "Cocoa Puff" doesn't tell us anything about which pitch classes appear in the set. The term *major triad* again serves as a helpful analogy. Just as there are twelve possible transpositions of that object we call a major triad, there are twelve possible transpositions of a given set (tacitly putting aside the potential for transpositional symmetry at this stage). The label we are seeking for our set will brand it according to its interval content, not its pitch classes.

Students are already versed in each brand of equivalence explored thus far, and these three categories are sufficient to pull Sets A, B, and D into a single category. To account for Set C, however, we must invoke a fourth category: *inversional equivalence*. This type of equivalence declares as "the same" any two sets whose consecutive intervals reflect one another when read in opposite directions (low to high and vice versa). Inversional equivalence accounts for the observation about the Webern sets provided as Figure 3: whether the set places a minor second atop a minor third or a minor third atop a minor second, the set remains a Cocoa Puff.

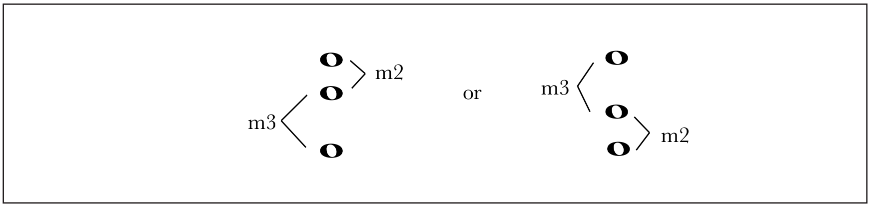

Figure 3 - Summary of intervallic content of sets from Figure 2.

[9] Many tonal theory textbooks appropriate the term *pitch class* to clarify fundamentals of pitch naming and notation as they are introduced, though explicitly outlining its relationship to octave and enharmonic equivalences is essential to this introduction to pitch-class set theory.

Set theory's employment of inversional equivalence is more difficult for many students to grasp at first. The conceptual difficulties reside both in terminological overlap and with the lack of correlation to tonal theory (not to mention the difference in sound between a given nonsymmetrical set and its inversion—this becomes especially apparent to students when they learn that major and minor triads are inversionally equivalent). By the time most curricula reach the topic of pitch-class set theory, students have been required to become conversant with intervallic inversion, triadic inversion, melodic inversion, and textural inversion. The meanings of these applications of inversion overlap in ways that sometimes perplex students. (This is to say nothing of row inversion, a crucial component of serialism that is often introduced shortly after pitch-class set theory.) Yet none of these applications of *inversion* is precisely the same as reversing the sequence of intervals in a temporally unordered set.[10] Some students are understandably bewildered when asked to think of inversion in one more slightly different way.

After briefly introducing the concept of inversional equivalence and demonstrating how it is needed if we are eventually to give Set C the same label as the others in Figure 2, I leave it aside for the moment and gently deflect the questions and objections that frequently arise. The power of this category of equivalence becomes clearer when we use it to generate a meaningful label for the set type we've been calling Cocoa Puffs.

Pyrophobic Architecture, Normal Order, and Prime Form

As I explain to students, the problem with an arbitrary label for our set type such as "Cocoa Puff" is that it is arbitrary. We cannot reasonably expect another analyst to independently come up with the same label, nor could we expect her to know what we mean by

[10] This definition of inversion holds up in pitch space and pitch-class space. Once a starting point is selected—whether a pitch in pitch space or a node in pc space—and the intervals of the original set are rewritten in the opposite order from that point, the set has been inverted. The careful and limited conflation of pitch space and pitch-class space implied in this definition of *inversion* is required because all operations will be illustrated to students in staff notation, i.e., in pitch space. Such conflation is virtually transparent to students as they progress from staff notation toward integers and clock-face notation.

this term unless we stop to explain ourselves. The utility of a label like "C-minor triad" is that the musical community has reached a consensus on what this label connotes, and its meaning can be recomputed independently. We need a label with similar utility for our set type.

To that end, I provide students a three-step process, reinforced on the blackboard or in a handout, to create such a label. The entire process is summarized (as I might in a handout) in Figure 4. I inform students that, for the moment, I am "skipping" step 2 (comparison of the set's normal order with the normal order of its inversion). Skipping this step gives students two advantages: they get to view a final product of the analytic process sooner and more easily, and they are clued in that step 2 will require some special conceptual effort.[11]

Step 1: Rearrange the set into normal order.
Make the building as short/safe as possible. Get all the notes into the same octave, with the smallest possible interval from lowest to highest note. If there's a tie for "shortest building," then pick the building that's safest for the CEO. If there's *still* a tie, then pick the building that's safest for the vice-president (and so on).

Step 2: Compare the normal order of the set with the normal order of its inversion.
Invert the set by writing its intervals backwards (placing the highest interval of normal order at the bottom of the inversion and so on). Check to see that the inversion is itself in normal order. Then, compare the normal order of the original set with the normal order of its inversion: which building is safer? (Which is the "m" and which is the "w?") Use the safest possible building for step 3.

Step 3: Label the set with its prime form.
Count the number of half steps each note is above the first note. Prime form labels always begin with "0" (the first note is always zero half steps above itself!).

Figure 4 - A three-step process for determining (Rahn's) prime form.

[11] When teaching via this approach I tell students, in as many words, "We're skipping step 2 because it deals with inversional equivalence, which is a little tougher for us to wrap our heads around. Let's get to step 3 so you can see the finished product, and that will help us understand how step 2 works. In the meantime, you should know that—for the moment—I'm cherry-picking sets that make step 2 moot." When put this way, students have no trouble negotiating the procedure for finding prime form.

Step 1: Rearrange the set into normal order. I ask the class members to imagine they are architects with hypersensitivities to fire safety. Their goal is to design a building (i.e., rearrange the notes of the set using octave equivalence) so that it is as short as possible (i.e., exhibits the smallest possible interval from lowest to highest note), thus allowing people on the highest floors of the building to exit quickly. This arrangement, known as *normal order*, is analogous to mentally stacking a complex triadic structure in root position so as to determine its makeup.

After pointing out that the class has already instinctively put the Webern sets in normal order (as shown in Figure 2), I ask students to try doing the same for the sets shown in Figure 5. Finding normal orders for Sets E and F presents no special difficulty. These sets also allow me to introduce the trick of writing the set's pitch classes into a single octave (duplicating at the octave whatever note I arbitrarily select as lowest), locating the largest interval between consecutive notes, and using the higher note of the interval as the lowest note of normal order. Determining normal order for Set G presents a new complication; as shown in Figure 5, there are two orderings of this set that produce a "shortest" building. Extending our fanciful architecture metaphor can help us determine normal order in this case. If (and only if) there's a tie for shortest possible building, our pyrophobic architect then selects the design providing the CEO in the penthouse—represented as the second note from the top of the registral ordering—the best chance of getting out in the event of a fire. In the case of the two orderings of Set G shown in Figure 5, the penultimate note of order 1 is a minor third away from the "ceiling" while the penultimate note of order 2 is only a major second away, so we prefer the "safer" order 1. In the event that the two CEOs are equidistant from their exits, we then compare the "safety" of the next-lowest notes in the set (the "vice-presidents," perhaps?), and so on.

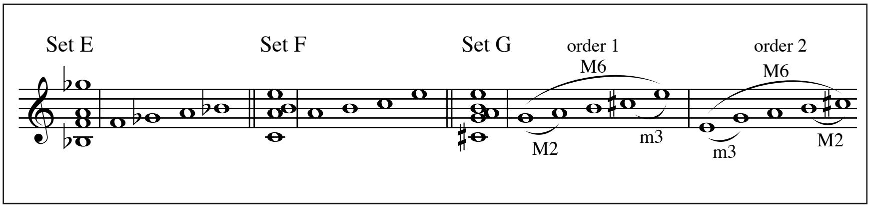

Figure 5 - Three sets demonstrating normal order.

Note that this approach to normal order reflects that of John Rahn's *Basic Atonal Theory*, adopted by many recent textbooks, which provides different results for a small number of set classes from those originally presented by Allen Forte in *The Structure of Atonal Music*.[12] If the instructor prefers that students eventually arrive at Forte's prime-form labels, they should be instructed to favor the safety of "the first-floor receptionist" (i.e., the second note from the bottom of the registral ordering) rather than that of the CEO. For Set G the result is the same—order 1 shows the receptionist a major second away from the bottom of the building, a half-step "safer" than order 2's receptionist.

Step 3 (remind students that we're skipping step 2 for the moment): *label the set with its prime form*. For the sets we've encountered thus far, this is easily accomplished by examining the normal order and counting the number of half steps each note is above the first note. Set E's prime form is (0145), Set F's prime form is (0237), and Set G's prime form—derived from order 1—is (02469). Note that these sets were carefully pre-selected so as to momentarily render moot the issue of inversional equivalence. Students will learn when we introduce step 2 of our three-step process that this issue requires further testing of the set's normal order before we can apply a prime-form label with certainty. Briefly leaving aside inversional equivalence allows students to see the goal product of this process without simultaneously needing to attend to this complication.

It is worth pointing out to students that once the prime form label has been determined, it applies to any manifestation of the

[12]John Rahn, *Basic Atonal Theory* (New York: Schirmer, 1980), 33 ff.; Allen Forte, *The Structure of Atonal Music* (New Haven, CT: Yale University Press, 1973), 3–5. Forte's treatment of normal order is derived from Milton Babbitt, "Set Structure as a Compositional Determinant," *Journal of Music Theory* 5, no. 1 (1961): 72–94, reprinted in *The Collected Essays of Milton Babbitt*, ed. Stephen Peles (Princeton: Princeton University Press, 2003). Texts employing Rahn's approach to normal order include the sixth edition of Stefan Kostka and Dorothy Payne's *Tonal Harmony with an Introduction to Twentieth-Century Music* (New York: McGraw-Hill, 2009), the third edition of Kostka's *Materials and Techniques of Twentieth-Century Music*, the third edition of Straus's *Introduction to Post-Tonal Theory,* Roig-Francolí's *Understanding Post-Tonal Music*, and J. Kent Williams's *Theories and Analyses of Twentieth-Century Music* (Fort Worth, TX: Harcourt Brace, 1997). In contrast, Jane Clendinning and Elizabeth Marvin's textbook *The Musician's Guide to Theory and Analysis* (New York: W.W. Norton, 2005) uses Forte's approach. See also note 13.

same set in any registral ordering. For instance, the prime form for Set G's order 2 is *also* (02469): despite the reordering of its pitches, the same label still applies. A compelling way to illustrate this fact is to provide students with a list of set classes (available in many textbooks[13]). If we derive a "prime form" from the arrangement of intervals that constitutes order 2, we arrive at (03579). This "set class" doesn't appear in any set-class list because there's no such thing as a (03579) set type; as we've already established, this is a registral re-ordering of (02469). (In the same way, a major triad remains a major triad even when its third appears in the bass. The label major triad, like prime form labels, makes use of octave equivalence and thus does not depend upon registral distribution for its identity.)

We have now erected enough of a logical set-labeling system for students to recognize the objective of our method: a numerical label that accounts for the number of half-steps from one note of the set to each other note in the set when the set is arranged in a particular registral order. Understanding the structure behind the label we intend to derive will aid students in assimilating the concept and consequences of inversional equivalence.

Chocolate Candies and Inversional Equivalence

When we return to the Webern sets of Figure 2, students quickly realize that our labeling system is not yet rigorous enough to provide a single rational name for Sets A through D. Set C takes the prime-form label (014), but the other three sets each appear to work out to (034), another "false" prime form that doesn't appear in any set-class list. The problem is clear: Sets A, B, and D are (transposed) inversions of Set C. As I remind students, the goal of our theory is to provide a single label for all four of these sets given their identical intervallic construction. Apparently, the label we will favor is (014), since the set-class list includes this label and not (034). According to

[13] I allow students to refer to set-class lists to check their work as they begin finding prime form labels, but warn them that they will soon be expected to identify set classes correctly without access to a list. Set class lists appear in Clendinning and Marvin, A85–A87; Kostka, 319–22; Kostka and Payne, 597–600; Roig-Francolí, 362–65; Straus, *Introduction to Post-Tonal Theory*, 261–64; and Williams, 337–39. The *fifth* edition of Kostka and Payne's *Tonal Harmony* (2004, 557–60) includes a pitch-class set catalog that shows prime forms as both Forte and Rahn would label them.

the property of inversional equivalence, then, all four of these sets *should* take the label (014)—there is no such thing as an (034).

M&M candies can lucidly and whimsically ingrain the principle of inversional equivalence. In this approach, students are each asked to place two candies on a desk, with one oriented so that the printed "m" faces the student and the other rotated so that it resembles a "w." Asked, tongue-in-cheek, "How many 'm's are on your desk?" students acknowledge that there are two; one is simply upside-down. In the same way, the set we might initially label (034) is in fact an "upside-down" (014)—that is, (034) is a "w," and the "m" is (014). We have been conditioned by a candy company, Mars Incorporated, to recognize the printed letters on their candies as ms, no matter their orientation. Similarly, there exists a convention (developed by Forte and refined by Rahn) that allows us to favor one "orientation" of each set class to stand as a label for all its manifestations.

Learning how to apply this convention involves the missing step 2 from our three-step labeling method already explored. Consider Set H in Figure 6. Step 1 of our labeling method, placing the set in normal order, is already accomplished in the figure. Step 2, which we omitted previously, is to *compare the normal order of the original set with the normal order of its inversion*. Figure 6 illustrates one relatively quick way to realize the inversion: start on the lowest note of the normal order already written (in this case, A) and write out the ascending intervals of normal order backwards. The highest/last interval of Set H's normal order is a major second, so the lowest/first interval of its inversion will be a major second; the penultimate interval of normal order is a minor second, so the second interval of the inversion will be a minor second, and so on. When inverted correctly, the resulting set should start and end on the same notes as the original normal order—this constitutes an easy way to check one's work.

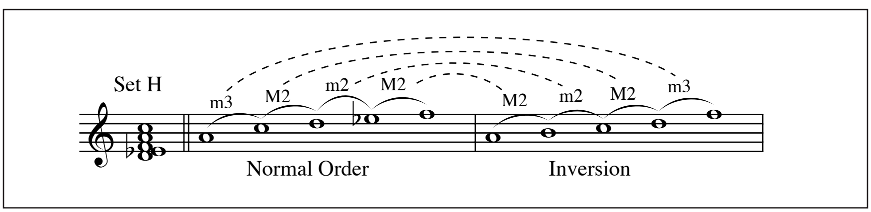

Figure 6 - Comparison of a set with its inversion.

A student objection that occasionally emerges at this point is that, in inverting this set, we have changed its pitch-class content. This confusion might be articulated with the question, "How can the inversion be the same set? It has different notes in it." In an important sense, this student is correct: the set we have created by inverting Set H is a different set. The confusion results from failing to remember that the prime form label we are seeking to affix to Set H is based on its successive interval content, not its pitch-class content. By inverting the set we have preserved that successive interval content; it has only been registrally flipped "upside-down." To pull off this inversion we had to hang the intervals on different pitches, creating a new set, but doing so did not change the set *type* by which we are hoping to identify Set H. Our eventual label is meant to describe a set class (i.e., a group of sets) of which Set H *is* but one instance.[14] To find that label we need to compare Set H with other sets in that class, including H's inversion.

Once the set has been inverted, that inversion should be compared against the original normal order using the same pyrophobic architectural principles we employed in step 1. The two "buildings" in Figure 6 naturally exhibit the same overall height (a minor sixth); this will always be the case when comparing a set against its direct inversion. As before, we next consider the relative safety of the two buildings' CEOs, and find that the D of Set H's inversion is a half-step closer to the bottom than the E♭ of the original normal order. (Those who favor Forte's set-class types over Rahn's will ask their students to compare the C of normal order with the B of its inversion, but will still find the inverted form of the building safer and will come to the same set type.) As we complete step 3 (labeling the set with its prime form), we should therefore use this inverted form to determine the numbers of the label: (02358).

The crucial corollary of labeling Set H's inversion as (02358) is that Set H itself also takes the label (02358). Without performing

[14]Driving students towards this realization is a main motivation for showing sets *as* pitches (on a staff) rather than as integers. I want students to realize that a set's inversion does contain different pitches (pitch classes), and for them to wrestle through the logical and musical consequences of this fact. When beginning students are "allowed" to convert sets to numbers representing their intervallic content, they become less cognizant of the distinction between a given set and the label we're working to ascribe to it. More-abstract intervallic representations of sets and set classes will naturally follow this introduction to the theory.

step 2, we might easily conclude that the prime form of Set H is (03568). Invoking the M&M metaphor, step 2 showed us that Set H's normal order was in fact the "w" for this set, and so we found the "m" by turning the "w" upside down—that is, by inverting it. Figure 7 highlights this inversional relationship as I might at the blackboard, emphasizing the fact that "(03568)" is actually (02358) turned on its head.

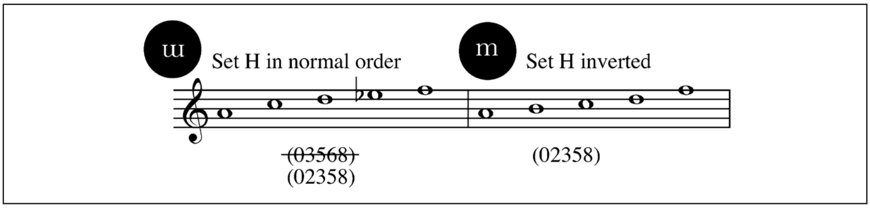

Figure 7 - Finding prime form of Set H.

Sets E, F, and G were selected because each is already an "m." Inverting any of these sets produces a less-safe building than the original normal orders (allowing us to temporarily skip step 2 at that stage of learning the theory). Looking back once more at the Webern sets, however, shows that we have now fleshed out enough of the theory to derive the same logical label for each of Sets A, B, C, and D. As Figure 8 shows, all four sets are instances of (014), and the "(034)" label we had tentatively assigned to three of these sets turns out to be the "w." "Why not label these sets as (034)?" a student might ask. After all, thus far in the Webern movement we've found more instances of this set type arranged with the half-step at the top of normal order rather than at the bottom. The answer to this question, of course, is established convention. Just as convention arbitrarily dictates the names we ascribe to the three varieties of augmented sixth chords, so we are constrained by the conventions invented by Forte and Rahn that determine how we label set classes. This constraint is hardly a weakness of the theory—without it, we wouldn't be able to agree on a single prime-form label for all instances of a set class.

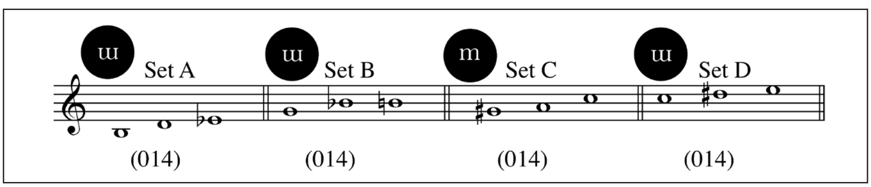

Figure 8 - Labeling the sets from Webern, Op. 5, No. 3.

Promoting Fluency and Demonstrating Relevance

Now that students are acquainted with the fundamentals of labeling a set with its prime form, they are equipped to research the significance of a single set type—now identified as set type (014)—to the musical landscape of Webern's Op. 5, No. 3. Figure 1 shows the many additional instances of this set class in mm. 1–10. Measure 6's dependence upon (014) is especially noteworthy in that the harmonic pizzicato (014)s are themselves planed to create melodic (014)s in each participating instrument. This set type also serves as the source of a cello ostinato that underpins the second half of the movement, as illustrated in Figure 9.

Figure 9 - Webern, Op. 5, No. 3, cello ostinato beginning at m. 15.

This analysis leads naturally to a discussion of segmentation: the selection of pitches from the score to form what the analyst believes to be a musically relevant set. The selections of the notes of Sets A through D in mm. 1–3 were virtually transparent because these pitches "belong together" rhythmically and timbrally. Measure 6 illustrates that sets may be presented horizontally as well. This music also allows the instructor to foreground the fact that the line between appropriate segmentation and gerrymandering is a subjective one. Is it significant to note that the cello C# and the first two melodic notes of violin 1 (D and B♭) also form an instance of (014)? Does it matter that those three pitch classes become more closely united as the head motive of violin 1's melody in m. 9? Students need not always agree about the appropriateness of a given segmentation, as long as they are aware of the temptation to justify questionable segmentations so as to fit an analytic conclusion they're hoping to find.

When students segment this movement looking for (014)s, they inevitably find other trichord types lurking as well. Set class (015) seems especially to play a prominent role alongside (014). The miniature canon of mm. 5–6 consists of melodic (015)s, as does the inversion canon between violin 1 and cello in mm. 7–8.

(In fact, any three adjacent notes in either of the inversion canon's melodic lines constitute an instance of (015).) (015) is emphasized again in the isolated cello motive of m. 8, and violin 2 uses it to conclude the section ending at m. 14. As students begin to realize the dual importance of (014) and (015) to this movement, the instructor might draw their attention to the violin 1 melody of mm. 9–10, which returns to provide the climactic unison ending to the movement shown in Figure 10. This pivotal melody fuses (014) and (015) with (013), the trichord type that most emphasizes the interval type shared by (014) and (015). The movement's ultimate C# is also accounted for in this analysis, as it completes a final statement of (014) that overlaps with the melody's (015). Even if Webern never labeled the building blocks of this movement in the manner provided by this theory, students' analytic work makes clear that these trichord types are essential to understanding how the music is constructed. Moreover, this analytic exercise provides students' practice in segmentation and in quickly finding prime form for a given trichord, while demonstrating how this theory offers an appropriate method for investigation of atonal music.

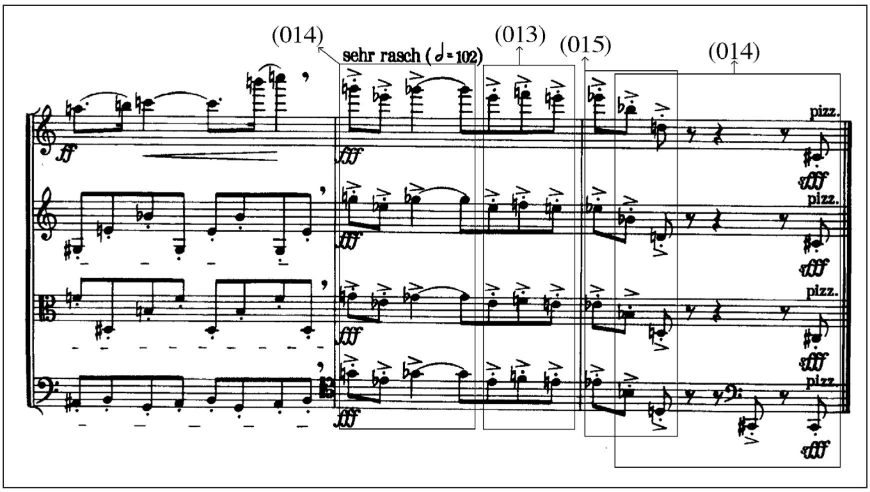

Figure 10 - Webern, Op. 5, No. 3, conclusion (mm. 21-23) with analysis.

A next step in building fluency in the theory is analysis of the sets provided in Figure 11, each of which demonstrates a special quality. Set I, when placed in normal order, replicates itself under inversion—that is, its interval read the same forwards or backwards (m2/m3/m2). Such a set is *inversionally symmetrical*. Set J is presented as a measured melody to remind students that sets can be harmonic or melodic units. This set, shown in normal order, is *transpositionally symmetrical*: when transposed a certain distance (in this case a tritone), the set reproduces its pitch classes as illustrated by the slurs. Of course, a great many set classes exhibit both transpositional and inversional symmetry—(0167) is a famous example. As an instructor, I find it helpful to first display set classes that exemplify only one brand of symmetry, but I then challenge students to think of set classes that demonstrate both types: "What triad type is both inversionally and transpositionally symmetrical? Which traditional seventh chord exhibits both kinds of symmetry?"

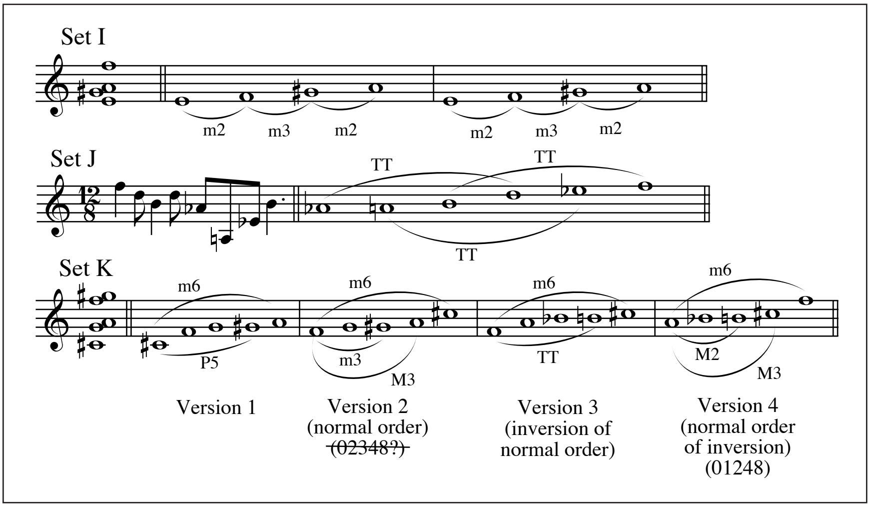

Figure 11 - Three sets analyzed.

Set K presents an additional challenge, and I therefore usually save this final difficulty in set-class labeling for another day of instruction. Students readily determine that there exists a tie for "shortest building," but settle on version 2 as the set's normal order because of its smaller distance between the CEO and the front door (i.e., a major third rather than a perfect fifth from lowest note to next-to-highest note). Version 2's direct inversion also holds the CEO further from the front door (compare version 3's tritone versus version 2's major third), leading us to believe that we should derive the prime form label from the set's original normal order. The trouble is that the label we pull from version 2, (02348), is nowhere to be found in our set-class list. Where did we go wrong?

The answer lies in the fact that version 3 itself is not yet in normal order. Put another (seemingly paradoxical) way, the inversion of normal order is not the normal order of the inversion. Version 4 of Set K places version 3 in normal order. The CEO in this final manifestation ties the CEO of version 2 (both exhibit a major third between the lowest and next-to-highest notes), but also gets its "vice-president" a half step closer to the front door (a major second versus a minor third). The prime form version 4 suggests, (01248), is validated by a set-class list and thus represents every version of Set K presented in Figure 11.

The complication represented by Set K occurs only in sets that exhibit a tie for "shortest building" (and are not inversionally symmetrical), so students will be alerted to this issue as they begin working with a given set.[15] If the "shortest building" tie doesn't hold, then the normal order's direct inversion will be itself in normal order. In this case, students can quickly visualize the inversion by comparing the highest and lowest intervals of the original normal order. Reconsider Set H in Figure 7. When written in normal order, we can see immediately that the "CEO interval" from A to E♭ is larger than the CEO interval will be in the inversion (represented in the original normal order's interval from C up to F). Without writing out the inversion, we have determined that the original form of the set is the "w," and we can derive prime form—(02358)—by calculating intervals downward from the F at the top of the original normal order (F down to E♭ is two half steps; F down to D is three half steps, and so on). But in the event that the original set demonstrates a "shortest building" tie, the simplest advice to offer students is to explicitly work out the inversion's normal order before selecting the "safest" building.

[15]Other sets that may serve as examples of this problem include {A C C# F}, which is a member of the (0148) class (and not "(0348)" as it may first appear), and {B C# D F G#}, a member of the (01369) class (rather than the apparent "(02369)" class). Students may also find it reassuring to learn that, among set types of five elements or fewer, this issue obtains in only one tetrachord—(0148)—and only four pentachords: (01248), (01568) (this set type is labeled (01378) by Forte), (01369), and (01469).

Next Steps for Exploration

I have found it helpful to delay students' first reading in atonal set theory until its basic tenets have been introduced in class as described above. Because of its unfamiliarity and its complexities, students often struggle simply to complete an initial reading on this theory if the instructor has not yet introduced it. Once students are already able to put a set in normal order and identify its prime form label, however, several texts can provide appropriate reinforcement and further development of the theory. I most often assign the "Nonserial Atonality" chapter from Kostka's *Materials and Techniques of Twentieth Century Music*; as noted above, the in-class approach described in this essay is indebted to Kostka's own treatment of this topic. This chapter also introduces interval classes and interval class vectors, Forte labels, inclusion and Z relationships, and aggregate completion as a compositional strategy. Other recent theory textbooks largely devoted to tonal music also provide introductions to set theory as summarized in Figure 12. When choosing a supplementary reading, instructors should consider carefully the speed with which they expect students to replace with integers and mathematic abstractions the principles they have now conceptualized using musical notation, as well as the extent to which they intend to immerse students in additional aspects of the theory (Forte numbers, complementation, inclusion relations, T_nI, transpositional combination, voice-leading between sets, etc.).

In addition to probing interval class vectors and inclusion relationships, in my program students must aurally recognize and sing on demand any trichord in normal order. Besides stretching students' aural skills towards the exotic flavors of atonal music, this requirement leads students to intimate familiarity with at least one cardinality of set classes and reinforces the principles of set-class identity. Being asked to sing both the "m" and "w" versions of (016), for instance, reminds students that this set class can exhibit a half step either at the top or the bottom of normal order, and that this dissonant-sounding trichord does contain a perfect consonance.

As with the in-class introduction of the theory itself, the first analytic assignments should also ask students to bring to light insights only available through application of the theory. The assignment I use, based on the fourth movement of Webern's Op. 5, asks students to identify inclusion relationships among musically significant set classes and to recognize a particular septachord type

Tonal Theory Texts	Clendinning and Marvin, *The Musician's Guide to Theory and Analysis* (chapters 30–32)	Demonstrates both integer notation and musical notation to find prime form, introduces clock faces, T_nI, ic vectors, and set complementation
	Kostka and Payne, *Tonal Harmony*, 6th ed. ("Post-Tonal Theory")	Uses musical notation to find normal order; introduces integer notation, clock faces, and T_nI; mentions ic vectors but does not explain how to derive one
	Turek, *Theory for Today's Musician* ("Twentieth-Century Techniques")	Uses musical notation to find normal and prime form, no clock faces
Post-Tonal Theory Texts	Kostka, *Materials and Techniques of Twentieth-Century Music,* 3d ed. ("Nonserial Atonality")	Uses musical notation to find normal order and prime form; introduces ic vectors, no clock faces
	Roig-Francolí, *Understanding Post-Tonal Music* (chapters 1, 3, and 4)	Favors integer notation and clock faces to determine normal order and prime form, introduces T_nI, ic vectors, set complementation, inclusion relations
	Straus, *Introduction to Post-Tonal Theory* (3d ed.)	Uses integer notation and clock faces to determine normal order and prime form, introduces T_nI, ic vectors, set complementation, inclusion relations

Figure 12 - Treatment of some basic principles of pitch-class set theory by selected textbooks.

as a cadential, form-defining element to this piece. Discovering and contrasting the "favorite" set classes of Bartók and Schoenberg, and later identifying set classes' role in certain brands of serialism, also reinforce the musical and analytic value of this theoretical apparatus. While sterile drill in labeling prime forms may be required to develop fluency, the theory's musical relevance must be demonstrated again and again in guided and prescribed analyses if students are to "buy into" the theory's value.

Finally, instructors might consider illustrating this theory's flexibility and utility in treating music not typically described as atonal. Finding specific set types used with internal consistently in pitch-centric works helps reinforce the theory and its wider relevance. Roig-Francolí actually introduces this theory using Debussy's "La cathédrale engloutie," using it to demonstrate how the prelude coheres through motives that focus on (025), (027), and (013).[16] Stravinsky and Bartók make use of transpositional combination of tetrachords to generate octatonic collections in the Symphony of Psalms and various *Mikrokosmos* pieces respectively.

[16]Roig-Francolí, 15–17.

At the least, set-class labels provide a convenient method of identifying important harmonies or melodic segments in all kinds of post-tonal music, and the act of labeling such musical elements in this way can draw the analyst's attention to intervallic relationships and connections that might otherwise be missed.

Conclusion

The goal of this essay has been to demonstrate how pitch-class set theory might be introduced in a typical undergraduate curriculum so as to convince students of the value of the theory itself and to nurture facility with its fundamentals. Nestled into a four- or five-semester sequence, this topic might only be treated for two or three class meetings before the instructor is compelled to move on to serialism, post-tonal pitch centricity, or other essential topics; even so, subsequent analytic work may continue to make use of this theory as appropriate. Given several weeks or an entire semester to devote to this topic, this introduction could be followed by a greater exposure to numerical depiction of pitch classes. The abstraction of operations like transposition and inversion to mathematical representations can prove useful and powerful to students. Such abstraction opens the door to extensions of the theory such as similarity relations, pitch-class set genera, and transformational ("Klumpenhouwer") networks.

One of my aspirations as an instructor of music theory is to equip students to deal responsibly with any repertoire they may encounter as professional musicians and disseminators of music. Another is to immerse students in unfamiliar music with analytic tools that allow them to ascertain and describe its features, thus broadening their tastes and increasing their appetites for new aesthetic experiences. I find that working with pitch-class set theory in the ways described above meets these goals in a way that enhances the myriad musical and analytical encounters (using this theory and others) I hope to provide my students each term.

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