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The History of Music Theory and the Undergraduate Curriculum

Robert W. Wason

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ROBERT W. WASON

Since the 1950s we have made tremendous progress in our knowledge of the history of music theory; yet, apparently very little has filtered down to undergraduate textbooks. The present paper shows that other fields—particularly philosophy and mathematics, whose histories are intertwined with ours—make much more use of their own history in undergraduate education. The paper then provides a view of a historically and theoretically based curriculum that Matthew Brown and R. Wason taught at the Eastman School in 1999-2000. It finishes with a call for a "liberal book" on music theory for undergraduates, as one writer on science education has called books that provide a richer intellectual context for the scientific skills they teach (Michael R. Matthews, *Time for Science Education: How Teaching the History and Philosophy of Pendulum Motion Can Contribute to Science Literacy* [NY: Kluwer/Plenum, 2000]). That author contrasts this to "professional texts," which "lack a story line: concepts, definitions, refinements, model problems and end-of-chapter exercises are the staple" (323)—an apt description of many undergraduate music theory textbooks as well.

I.

The years since the end of World War II have seen remarkable growth in our knowledge of the history of music theory. One thinks, for example, of Ratner's articles on 18th-century theory from the late 1940s and 50s, Strunk's *Source Readings* of 1950, or certain articles in the newly conceived encyclopedia, *MGG*, such as Palisca's survey, "Kontrapunkt," published in 1958.[1] The pace picked up considerably in the 60s through 2000, with the appearance of at least five historical translation series[2] and many groundbreaking articles and books, the latter including SMT prize-winners.[3] The

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This article originated as a keynote lecture for the forty-fifth annual meeting of the Music Theory Society of New York State (April 2-3, 2016), held at the Mannes School of Music at the New School. This version reflects the occasional nature of the work's origins, and the original lecture with voiceover can be found as a PowerPoint file at this journal's website: http://jmtp.appstate.com/articles.

1 Ratner (1949 and 1956), Strunk (1950), and Blume (1949-).

2 American Institute of Musicology, Armen Carapetyan, Director; Yale University Press Translation Series, Claude V. Palisca, ed.; Brooklyn, NY and Ottawa, Canada: Institute of Mediaeval Music, Musical Theorists in Translation; Colorado College Music Press Translations, Albert Seay, ed.; University of Nebraska Press, Greek and Latin Music Theory, Thomas J. Mathiesen, ed.

3 I note that 2018 was a banner year for the history of theory, with *two* SMT prize winners: Hicks

2002 *Cambridge History of Western Music Theory*[4] was essentially a stocktaking of this development, greatly aided by the work carried out at the Prussian State Institute for Music Research in Berlin. The Berlin *Geschichte der Musiktheorie* commenced publication in 1984; nine volumes were available by the mid-'90s when the Cambridge history got underway.[5] There's still plenty to do in the history of music theory. Much of the virgin territory provisionally mapped by Ian Bent in an SMT keynote address nearly a quarter of a century ago remains unexplored.[6] Still, my teaching of the early history of theory since about 1990 has shown me that the story already revealed is a very long, rich, and interesting one.

In fall 2015, for example, I began my course by talking about music theory in cuneiform writing, carved with a wedge-tipped reed on clay tablets in ancient Mesopotamia. In Example 1 we see, from a number of different angles, a relatively late tablet dating anywhere from the Kassite to the Neo-Babylonian period, c. 1600 to 530 BCE, the most recent date contemporaneous with Pythagoras. The chronology is given in Example 2.

CBS 10996 was of interest to scholars first for its mathematical content, but later was found to contain music theory as well, demonstrating the close relationship between them. Once an earlier tablet was deciphered, it was possible to reconstruct the names and placement of fourteen intervals: the seven possible scalar fifths/

Example 1
Tablet CBS 10996, at the Museum of the University of Pennsylvania.

(2017), and Parkhurst (2017).

4 Christensen, ed. (2004 [2002]).

5 Zaminer, ed. (1984-2006). The projected total of fifteen has been revised: eleven volumes have appeared; the twelfth and last is apparently still underway.

6 Bent (1993).

fourths (counting the tritone) and the seven thirds/sixths, squeezed into a mod-7 octave as shown in Example 3. The interval names enabled scholars to read the tablet "Fragment 7/80," summarized in part in Example 4. Its harp tuning method amounts to a series of "modulating" scales that adumbrate the Greek *tonoi*. Thus the basic materials of Ancient Greek music theory very likely came from Mesopotamia.[7]

Music in ancient Mesopotamia and Egypt

Marcelle Duchesne-Guillemin

The civilization which began at Sumer developed contemporaneously with that of Ancient Egypt. As is well known, the latter was very much influenced, from the New Kingdom onward, by the Babylonian and Hittite off-shoots of the Sumerian culture. One of the most striking elements of Sumerian culture is the sudden appearance about 2600 B.C. of instruments (harps and lyres) so elaborate that they presuppose a long previous development of which no trace is left. Here it may be convenient to give some indication of the conventional chronology followed:

Mesopotamia		**Egypt**	
Prehistoric period	pre-3000 B.C.	*Predynastic*	pre-3000 B.C.
		First and second dynasties	3000–2780
Sumerian civilization		*Old Kingdom*	
Early dynastic	2900–2370	3rd–beginning 11th dynasties	2780–2040
Royal cemetery of Ur	2600	*Middle Kingdom*	
Sumero-Akkadian period	2370–2100	11th and 12th dynasties	2040–1730
3rd dynasty of Ur	2100–2000	Hyksos period	1730–1562
Babylonian civilization		*New Kingdom*	
Old Babylonian	2000–1600	18th dynasty	1562–1308
Kassite invasion and hegemony	1600	19th and 20th dynasties	1308–1085
Rise of Assyrian power	1350–1000	*Late period*	
Assyrian empire	1000–612	21st–26th dynasties	1085–525
Neo-Babylonian period	612–539		
Achamenid period	539–330		
Parthian period	250 B.C.–A.D. 330		
		Ptolemaic period	306–30 B.C.
		Roman period	1st century B.C.–6th century A.D.

World Archaeology Volume 12 No. 3 Musical instruments

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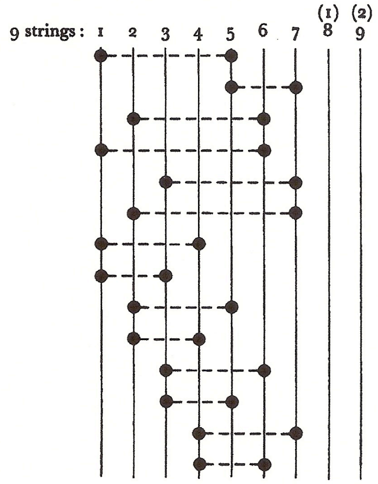
Example 2

Mesopotamian Chronology (from Duchesne-Guillemin 1981).

7 The best introduction to this topic is Kilmer and Mirelman 2001.

Fig. 1. Interval names according to Text 1 (CBS 10996)

1	1–5	*niš gabrî*	" rise of the duplicate "
a	7–5	*šēru*	" (main theme of a) song "
2	2–6	*išartu*	" normal "
b	1–6	*šalšatu*	" third "
3	3–7	*embūbu*	" reed pipe "
c	2–7	*rebūtu*	" fourth "
4	4–1	*nīd qabli*	" fall/place of the middle "
d	1–3	*isqu*	" throw stick/lot "
5	5–2	*qablītu*	" middle "
e	2–4	*titur qablītu*	" bridge of the middle "
6	6–3	*kitmu*	" closed "
f	3–5	*titur išartu*	" bridge of the normal "
7	7–4	*pītu*	" open "
g	4–6	*serdû*	" (name of a) lament "



Example 3
Mesopotamian interval names transcribed from CBS 10996 (from Kilmer 1984).

Well here's Pythagoras in Example 5, perhaps just back from Babylonia, where he is alleged to have traveled, carrying an anachronistic "book," of all things. It was the Ancient Greeks who put the theory into *Musica Theorica*,[8] for Pythagoras proposed a *structural explanation* of certain preferred intervals, i.e., *their ratios*, and started an unbroken tradition of music-theoretical thought that goes on to the present day.[9] I'd ask the authors of the undergraduate books telling us of Pythagoras's discovery to keep in mind, however, that an interval as a "ratio" is not immediately intuitive to most musicians. Surely Aristoxenus's competing idea that an interval is a distance along an imaginary line gets much closer to our musical intuition. The controversy surrounding these ideas remains worthy of discussion today, and as the first true "music theorist" with a name, Aristoxenus, who actually set ground-rules for music composition, surely should be given his due! Yet, admittedly, the *musical* conception

8 "[T]hat the methods worked was sufficient justification to the Babylonians for their continued use. The concept of proof, the notion of a logical structure based on principles warranting acceptance on one ground or another, and the consideration of such questions as under what conditions solutions to problems can exist, are not found in Babylonian mathematics." Kline (1990, v.1, 14).

9 See the catalogue of Phanes Press, David Fideler, editor. Today's practitioners are surely outliers, but they do exist.

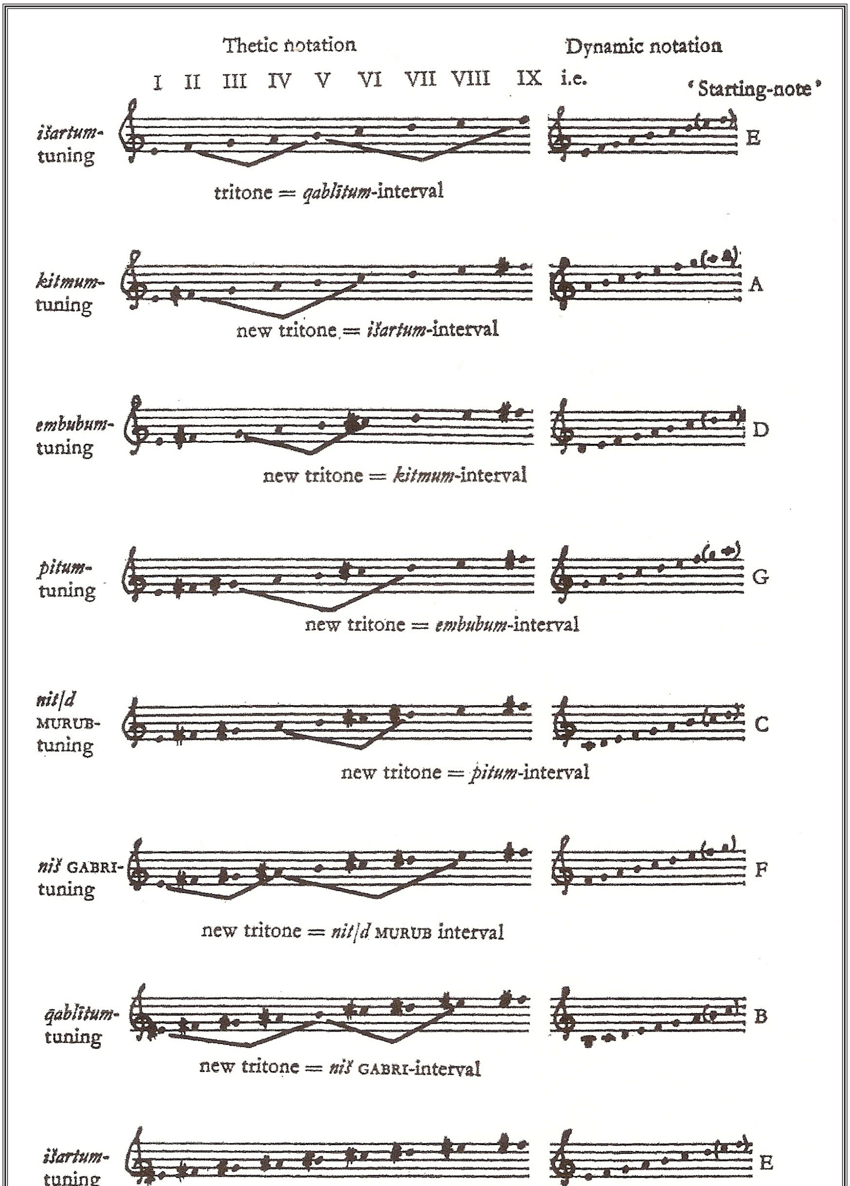
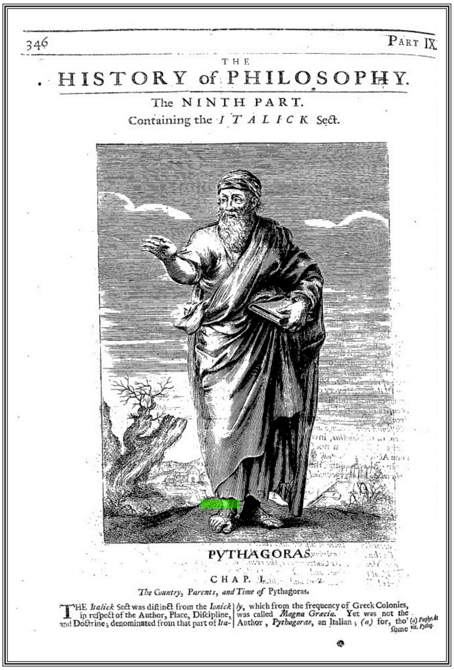


Fig. 1.

Example 4
Mesopotamian scales (from Wulstan 1968).

of "interval" was of secondary interest, especially during the Classical Period, for the musical ratios and proportions were regarded as instantiations of *harmonia*—the structure of the human soul, and of the very Universe itself. *That* was the point! The salubrious effect of music so constructed prompted Plato (on the left in Example 6) to make it foundational to education in his ideal republic.[10] Nine hundred years later, *musica*, the further development of these "universal" ratios and proportions by Pythagoreans, Neo-Pythagoreans and Neo-Platonists, became part of the curriculum dubbed *quadrivium* by Boethius (Example 7). Along with *arithmetica*, *geometria* and *astrologia*, and the preparatory curriculum later called *trivium*—*grammatica*, *rhetorica*, and *dialectica*—it was taken into the new universities in the 13th century (Example 8).



346 PART IX.

THE HISTORY of PHILOSOPHY.

The NINTH PART.

Containing the ITALICK Sect.

PYTHAGORAS.

CHAP. I.

The Country, Parents, and Time of Pythagoras.

THE *Italick* Sect was distinct from the *Ionick* in respect of the Author, Place, Discipline, and Doctrine; denominated from that part of *Italy*, which from the frequency of Greek Colonies, was called *Magna Græcia*. Yet was not the Author, *Pythagoras*, an Italian; (a) for, tho' some

Example 5

Pythagoras, from Stanley (1701).

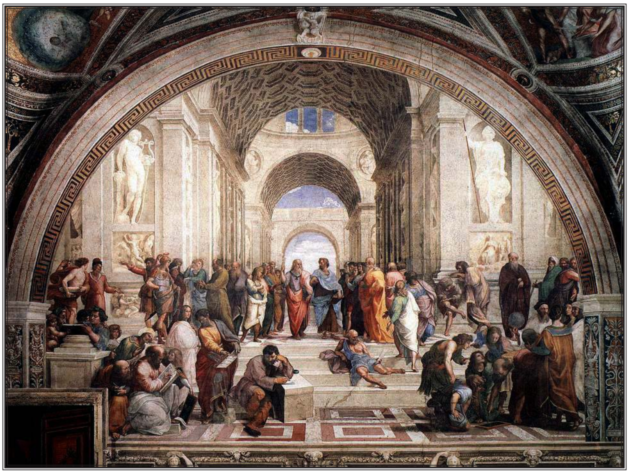
Miraculously, *musica* survived a withering critique during the Scientific Revolution of the 17th Century.[11] In Book I of his *Traité de l'Harmonie* of 1722 (Example 9) "Du rapport des Raisons & Proportions Harmoniques," Rameau returned to what I call the "Classical Theory of Harmony" of Pythagoras and Plato as transmitted by Zarlino (who revised it to include imperfect consonances). Rameau developed it further to include dissonances as well—quite an extraordinary step.[12] The possibility of "dissonant harmonies" had arrived!

My whirlwind tour winds up here with the advent of the Modern Theory of Harmony that most of you know well, and is meant to show that when compared either to the histories of other academic disciplines, or of Western Art Music of quite recent vintage that most of our colleagues specialize in, we're in very good shape.

10 See "Republic III," especially 401.d-402.a, on music and poetry as essential curricular components. Cooper (1997, 1038f).

11 Cohen (1984).

12 Rameau (1722).

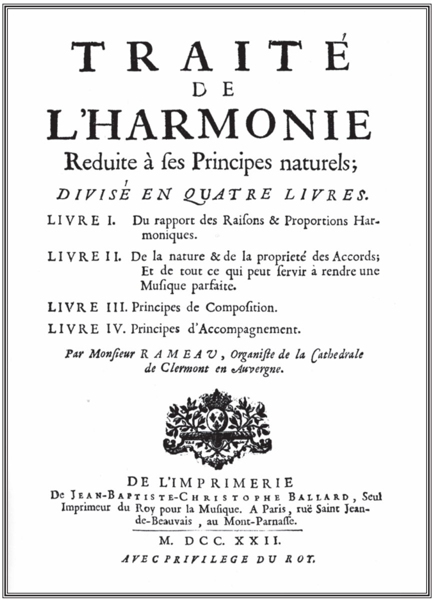


Example 6
Raphael's Scuola de Atene (1509-11);
Apostolic Palace, Vatican City.

Example 7
Boethius teaching his students. Initial from folio 4r of a ms. of Consolation of Philosophy
(Italy? 1385); MS Hunter 374 (V.1.11), Glasgow University Library.

Indeed, while *Philosophia* was the pinnacle of university study, its recorded history may be shorter than that of music theory, which likely arose from music making and teaching in Mesopotamia between 2000 and 3000 BCE, as we have seen. As the title

Example 8
The Trivium and Quadrivium. From Smits van Waesberghe (1969).



TRAITÉ
DE
L'HARMONIE
Reduite à ſes Principes naturels;
DIVISÉ EN QUATRE LIVRES.

LIVRE I. Du rapport des Raiſons & Proportions Harmoniques.

LIVRE II. De la nature & de la proprieté des Accords; Et de tout ce qui peut ſervir à rendre une Muſique parfaite.

LIVRE III. Principes de Compoſition.

LIVRE IV. Principes d'Accompagnement.

Par Monſieur RAMEAU, *Organiſte de la Cathedrale de Clermont en Auvergne.*

DE L'IMPRIMERIE
De Jean-Baptiste-Christophe Ballard, Seul Imprimeur du Roy pour la Muſique. A Paris, ruë Saint Jean-de-Beauvais, au Mont-Parnaſſe.

M. DCC. XXII.

AVEC PRIVILEGE DU ROY.

Example 9
Rameau's Traité de l'harmonie (1722).

of one of my undergraduate books put it, *History Begins at Sumer*."[13] Little did I know it at the time, but it's our history too as music theorists. Thus our story *should* be of interest to many who read about music theory, music in general, or cultural history—and to others reading in allied disciplines such as mathematics and language study, from which music theorists have long borrowed. Told well, it *can* be of interest to undergraduates too. Yet in my perusal of today's most-used theory textbooks, I didn't turn up much history of theory. True, old Pythagoras enters occasionally to introduce intervals, and Fux tries to motivate counterpoint within a discussion confined to harmony. And then there's Guido's hand..... But that's about it. And these appearances are almost never integrated into the larger narrative: they're usually in text boxes or appendices.

I wondered whether the situation was similar in other fields of study. On checking, I found that, unlike the history of theory at Eastman, the history of mathematics *is* taught in the undergraduate program at RIT and at my own University. And several authors—a couple even of trade books—have shown that new developments in mathematics and the sciences were sometimes prompted by the search for solutions to real-world problems.[14] The discussion of *musical* problems and the differences of approach of the various theorists dealing with them could make interesting reading as well, the most obvious cases in point being the varied reactions to the new tonal language of the eighteenth century and the musical languages of the twentieth. Perhaps there's an undergraduate music theory "reader" on tonality in that idea.

II.

I come now to Part IIA of this paper: a few telling remarks on two classic undergraduate theory textbooks. In the B section of Part II, I'll move to undergraduate textbooks in other subjects, for I chose a title that avoids the phrase "undergraduate curriculum in music" because I think that parts of our story can be told in synergetic courses co-taught by music theorists and specialists in other disciplines, and, most important, that undergraduate teaching in other disciplines may hold lessons for our own teaching. In Part III, I'll tell you about a core freshman course centered on theory, analysis and history of theory that Matthew Brown and I co-taught, and in Part IV, I'll attempt to answer two questions: 1. why has so little of the history of theory filtered down to undergraduate texts; and 2. what's to be gained by integrating some of it into those texts?

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13 Kramer (1959).

14 For example, see Kline (1953), Matthews (2000), and Sobel (1995).

As a representative of the Eastman School, I begin appropriately with one of Allen Irvine McHose's "green bibles," as Eastman students of the late 1940s through the 60s called them, *The Contrapuntal Harmonic Technique of the 18th Century*, published in 1947. Example 10 reproduces the cover of the "green bible"; Example 11 shows an interesting page that fits well with where I left off in my resume of the history of theory—Rameau, the practical theorist, with regard to whose method McHose has written (starting on the previous page):

> ...The practice of arranging the three notes of a triad in such a way as to have either the root, third, or fifth as the lowest note is called the *theory of inversion*...established by Jean Philippe Rameau during the 18th century. The student will find that this theory of inversion is the foundation approach to understanding the structure of music during the 18th and 19th centuries.
>
> It was Rameau's belief that major and minor keys were established by chord progression. His approach to proving this theory was through the theory of inversion. His method of attacking the problem confined itself to analysis of the music being composed by his contemporaries. His first step was to write, on a third staff placed below each line of the score, the root of every chord. The following Bach chorale illustrates Rameau's method of procedure: [See Ex. 11.]
>
> The succession of roots written on the accompanying staff below each line of composition is called the *fundamental bass* by Rameau. The next step in Rameau's analysis is to study the distance between each two bass notes in order as the composition progresses.[15]

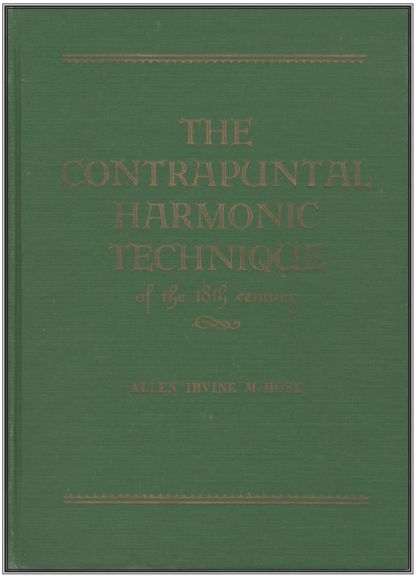
My first impression is that while not poetry, the meaning of these staccato, declarative sentences could not be clearer. At the same time, McHose follows good scholarly practice: he's also clear about just where he's coming from. Yet, when he claims that Rameau attempted to prove his theory by analysis of the music being composed by his contemporaries he exaggerates.[16] Rameau's work was hardly a "corpus study," as McHose advocates.

I've shown this particular page because of the Bach chorale with a fundamental bass. The young reader may get the impression that Rameau analyzed Bach, which is false.[17] Rameau, the most famous French composer of his time, was probably unfamiliar

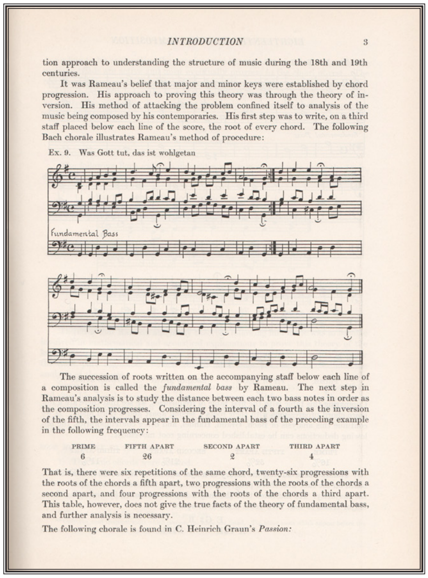
15 McHose (1947, 2-3).

16 There are perhaps a half dozen "analyses" in all of Jacobi's massive edition of Rameau's complete theoretical works (not yet available when McHose wrote, nor was Gossett's translation of the *Traité*). See Rameau (1971) and Erwin R. Jacobi, ed. (1967-72).

17 The analysis is not by Rameau. I double-checked with Thomas Christensen, *the* expert on Rameau, just to be sure I hadn't missed some new discovery. Perhaps because of McHose's book, I once heard a colleague at Eastman claim that "Rameau developed his theory to analyze Bach"—not a colleague in Theory or Musicology, I hasten to add.



Example 10
The "green bible": McHose 1947.



INTRODUCTION 3

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Ex. 9. Was Gott tut, das ist wohlgetan

The succession of roots written on the accompanying staff below each line of a composition is called the *fundamental bass* by Rameau. The next step in Rameau's analysis is to study the distance between each two bass notes in order as the composition progresses. Considering the interval of a fourth as the inversion of the fifth, the intervals appear in the fundamental bass of the preceding example in the following frequency:

PRIME	FIFTH APART	SECOND APART	THIRD APART
6	26	2	4

That is, there were six repetitions of the same chord, twenty-six progressions with the roots of the chords a fifth apart, two progressions with the roots of the chords a second apart, and four progressions with the roots of the chords a third apart. This table, however, does not give the true facts of the theory of fundamental bass, and further analysis is necessary.

The following chorale is found in C. Heinrich Graun's *Passion:*

Example 11
McHose on Rameau (McHose 1947: p. 3).

with some North German musician known principally as a virtuoso organist (though Bach certainly knew Rameau's music very well indeed). I *wish* Rameau had done this analysis; I'd like to see just how he would have handled the two stepwise progressions (questions McHose begs in his literalist interpretations), especially the second one. As is well-known, Rameau essentially forbade stepwise progressions, what might be called Rameau's law of chord progression, if we had music-theoretic laws named for their inventors as the sciences have. But he did *think* about them and develop clever work-arounds, and I doubt McHose has. Nonetheless, given the music theory of the time, McHose's book gets a pretty good grade in light of the present project, since he integrates his own work within the larger history of theory knowledgeably and convincingly.[18]

I now turn to a book that appeared fifteen years later, in 1962: Allen Forte's *Tonal Harmony in Concept and Practice*, a pioneering effort at adapting ideas drawn from 18th-century figured-bass theory and Schenker to the American pedagogy of harmony. When Forte's book first appeared, Schenker's *Harmony*, and Salzer's *Structural Hearing*,

18 He also recounts that history competently as he would have learned it from Shirlaw (1917), the authority of the time, whose book centers on Rameau.

were well-known.[19] But Forte never mentions Schenker's name until the second edition of 1974, when he includes an excellent chapter on "Large-Scale Arpeggiations, Passing and Auxiliary Notes," the opening page of which carries the footnote, "Although the influence of Heinrich Schenker is apparent in other portions of the present volume, it is most explicitly evident in this chapter. The author's indebtedness, which extends over a period of many years, is gratefully acknowledged."[20] Given the simplified voice-leading graphs Forte uses to make his points, as well as the chapter on "linear intervallic patterns" (a new idea in the second edition, inserted into the earlier chapter on "linear chords"), the acknowledgement is essential, but as Forte admits, the rest of the book––in the first edition as well, I'd add––shows the influence of Schenker, though primarily the influence of his later works, which were not translated at that point.

Despite his departure from conventional theory of the time, Forte is anxious to be seen as imparting a tradition, writing that "... many characteristics of the present textbook are deeply rooted in tradition as indicated by the passages quoted at the head of each chapter."[21] Let's look at the beginning of Ch. 1 as an illustration. Jean Benjamin de Laborde (a librettist, historian and composer—one of Rameau's students, in fact—) writes, presumably in Forte's translation:

> Composition consists in two things only. The first is the ordering and disposing of several sounds...in such a manner that their succession pleases the ear. This is what the Ancients called melody. The second is the rendering audible of two or more simultaneous sounds in such a manner that their combination is pleasant. This is what we call harmony, and it alone merits the name of composition.

Not surprisingly, Laborde sides with Rameau in the Rameau vs. Rousseau controversy. Rousseau took the anthropological view that melody reigned supreme, while Rameau was the theorist of "harmony" steeped in the tradition, though he would radically redefine it theoretically in both Book I and Book II of the *Traité*.[22] Yet Forte leaps to the end of the quotation: harmony = composition—case closed. What a missed opportunity to bring up an interesting debate, at least briefly, and to contextualize the study of harmony: melody *did* come first, of course, but it was *analyzed* harmonically—by the ancient Greeks!

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19 Schenker (1954); Salzer (1952).

20 Forte (1974, 397).

21 Forte (1962, iii).

22 For more on the controversy between Rousseau and Rameau, see Verba (1993, 38ff.).

As excellent as the core harmony-pedagogy is in Forte's book, he's a bit late in acknowledging the book's debt to Schenker and doesn't try hard enough to integrate the quotations from the history of theory into its overall narrative. In the book's defense, Appendix Two in the first edition provides a one-line biographical note and dates on each of the authors quoted, but inexplicably, it was removed from the second and third editions, and none of the entries gives us any indication as to *why* the writer in question was quoted. Thus, the quotations ultimately seem like 18th-century window-dressing. Confronting, in *greater depth*, Schenker, 18th-century history of theory—two topics that fit together rather well, after all—and their implications for the American teaching of harmony, would have improved the book.

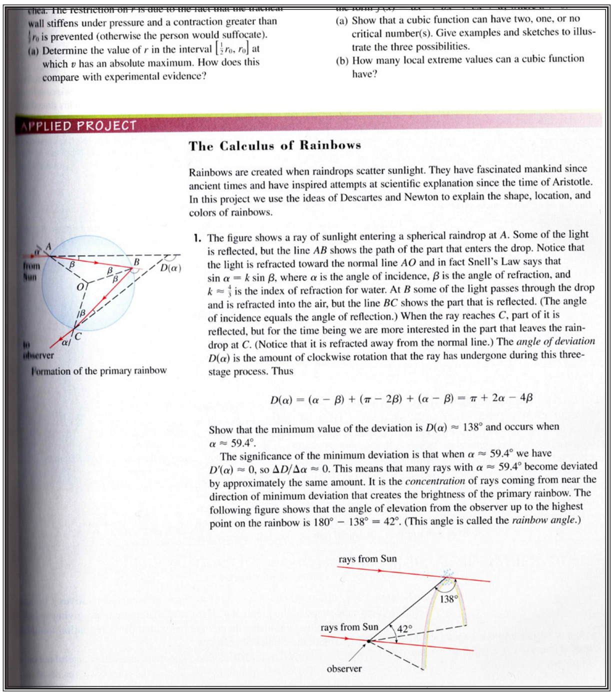
I turn now to undergraduate textbooks in five other subjects: mathematics, biology economics, psychology and philosophy. I emphasize at the outset that there's nothing comprehensive, or "scientific" about what I'm about to say: I worked completely ad hoc, largely from responses by colleagues in these fields I asked for "standard undergraduate textbooks."

I'll start with three books in mathematics, first the book on "contemporary abstract algebra."[23] Despite the title and relative youth of the discipline, the author writes that "...every undergraduate mathematics course should have a liberal arts character. I have tried to achieve this with comments, historical notes, quotations, biographies and photographs, and in general, by my approach to the entire subject" (Gallian 1990, xi). Biographies of 29 mathematicians are spread throughout the book, their ordering based on text topic rather than chronology. The book also includes a two-page "Index of Mathematicians," (Gallian 1990, A 39-40) listing, by my rough count, nearly one hundred names, often with multiple page references; those whose longer biographies are provided are bolded out. Of course, some of this historical approach is motivated by the general scientific practice of attributing solutions and innovations to those individuals who discovered or invented them. The seven-page "Index of Terms" (Gallian 1990, A-41-47) bears this out, but there's little overlap with the Index of Mathematicians, the page entries of which mark more than a mere mention of an eponymous inventor of a technique.

Next, I turn to James Stewart's calculus text of 2005. It's very long, colorful, and uses text boxes, pictures and marginal commentary frequently, presumably so that

23 Gallian (1990).

the text looks more inviting. In short, the book is a 21st-century computer-generated textbook, and weighs in like one, looking more like an encyclopedia volume (if you remember those), than a "book." It uses two devices rather well to contextualize the technical content: "applied projects" (e.g., "the calculus of rainbows," Example 12) and "writing projects" (e.g., "Newton, Leibnitz and the Invention of Calculus," Example 13); moreover, the marginalia occur frequently, and often consist of historical commentary. The project types and their clear ground-rules and bibliographies are particularly suggestive for us. "Applied projects" suggests compositional assignments, while "Writing Projects" suggests analytical or history-of-theory papers.



wall stiffens under pressure and a contraction greater than $\frac{1}{2}r_0$ is prevented (otherwise the person would suffocate).
(a) Determine the value of r in the interval $\left[\frac{1}{2}r_0, r_0\right]$ at which v has an absolute maximum. How does this compare with experimental evidence?

(a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.
(b) How many local extreme values can a cubic function have?

APPLIED PROJECT

The Calculus of Rainbows

Rainbows are created when raindrops scatter sunlight. They have fascinated mankind since ancient times and have inspired attempts at scientific explanation since the time of Aristotle. In this project we use the ideas of Descartes and Newton to explain the shape, location, and colors of rainbows.

Formation of the primary rainbow

1. The figure shows a ray of sunlight entering a spherical raindrop at A. Some of the light is reflected, but the line AB shows the path of the part that enters the drop. Notice that the light is refracted toward the normal line AO and in fact Snell's Law says that $\sin\alpha = k\sin\beta$, where α is the angle of incidence, β is the angle of refraction, and $k \approx \frac{4}{3}$ is the index of refraction for water. At B some of the light passes through the drop and is refracted into the air, but the line BC shows the part that is reflected. (The angle of incidence equals the angle of reflection.) When the ray reaches C, part of it is reflected, but for the time being we are more interested in the part that leaves the raindrop at C. (Notice that it is refracted away from the normal line.) The *angle of deviation* $D(\alpha)$ is the amount of clockwise rotation that the ray has undergone during this three-stage process. Thus

$$D(\alpha) = (\alpha - \beta) + (\pi - 2\beta) + (\alpha - \beta) = \pi + 2\alpha - 4\beta$$

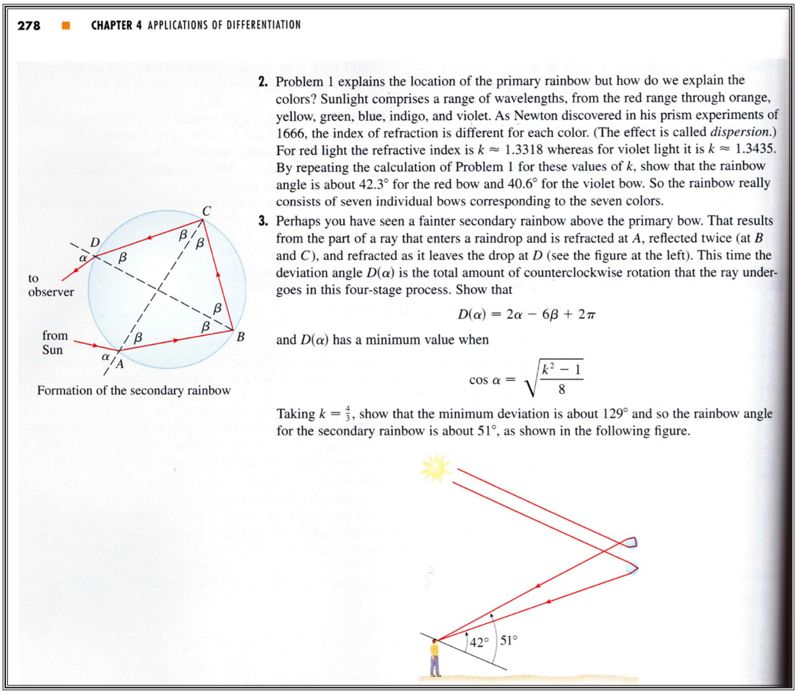
Show that the minimum value of the deviation is $D(\alpha) \approx 138°$ and occurs when $\alpha \approx 59.4°$.

The significance of the minimum deviation is that when $\alpha \approx 59.4°$ we have $D'(\alpha) \approx 0$, so $\Delta D/\Delta\alpha \approx 0$. This means that many rays with $\alpha \approx 59.4°$ become deviated by approximately the same amount. It is the *concentration* of rays coming from near the direction of minimum deviation that creates the brightness of the primary rainbow. The following figure shows that the angle of elevation from the observer up to the highest point on the rainbow is $180° - 138° = 42°$. (This angle is called the *rainbow angle*.)

Example 12
Applied project from Stewart (2005, 277).

Certainly the most relevant book to my project of any of the non-music ones was David Burton's 1980 book on number theory, the oldest book—and the oldest discipline—of the three.[24] In the preface, the author writes:

24 Burton (1980). The book remains in print in newer editions.


278 ■ CHAPTER 4 APPLICATIONS OF DIFFERENTIATION

2. Problem 1 explains the location of the primary rainbow but how do we explain the colors? Sunlight comprises a range of wavelengths, from the red range through orange, yellow, green, blue, indigo, and violet. As Newton discovered in his prism experiments of 1666, the index of refraction is different for each color. (The effect is called *dispersion*.) For red light the refractive index is $k \approx 1.3318$ whereas for violet light it is $k \approx 1.3435$. By repeating the calculation of Problem 1 for these values of k, show that the rainbow angle is about 42.3° for the red bow and 40.6° for the violet bow. So the rainbow really consists of seven individual bows corresponding to the seven colors.

3. Perhaps you have seen a fainter secondary rainbow above the primary bow. That results from the part of a ray that enters a raindrop and is refracted at A, reflected twice (at B and C), and refracted as it leaves the drop at D (see the figure at the left). This time the deviation angle $D(\alpha)$ is the total amount of counterclockwise rotation that the ray undergoes in this four-stage process. Show that

$$D(\alpha) = 2\alpha - 6\beta + 2\pi$$

and $D(\alpha)$ has a minimum value when

$$\cos \alpha = \sqrt{\frac{k^2 - 1}{8}}$$

Taking $k = \frac{4}{3}$, show that the minimum deviation is about 129° and so the rainbow angle for the secondary rainbow is about 51°, as shown in the following figure.

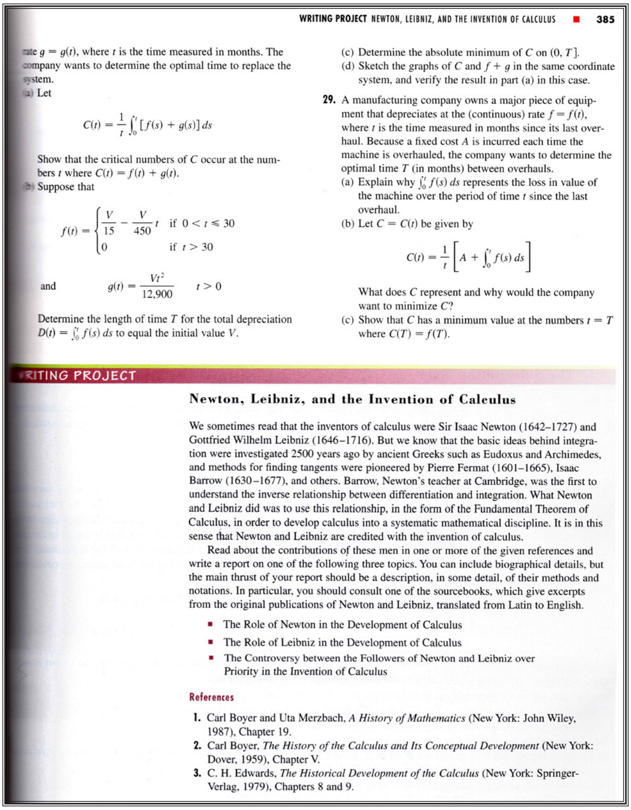
Formation of the secondary rainbow

Example 12 (cont'd)

Applied project from Stewart (2005, 278).

> The purpose of the present volume is to give a simple account of classical number theory, as well as to impart some of the historical background in which the subject evolved.... There is a dictum which says that anyone who desires to get to the root of a subject should first study its history. Endorsing this, we have taken pains to fit the material into the larger historical frame. In addition to enlivening the theoretical side of the text, the historical remarks woven into the presentation bring out the point that number theory is not a dead art, but a living one fed by the efforts of many practitioners. They reveal that the discipline developed bit by bit with the work of each individual contributor built upon the research of many others. (Burton 1980, v-vi)

Right after the Preface, we find the table of mathematicians who contributed to the theory, reproduced here as Examples 14 and 15. The chapters dealing with techniques beholden to a particular mathematician begin not just with a biographical summary, but with an installment in the history of number theory in which the mathematician figures prominently. Scan the first paragraph on Leonhard Euler in Example 16 and you'll get the idea.



WRITING PROJECT NEWTON, LEIBNIZ, AND THE INVENTION OF CALCULUS 385

rate $g = g(t)$, where t is the time measured in months. The company wants to determine the optimal time to replace the system.

(a) Let

$$C(t) = \frac{1}{t}\int_0^t [f(s) + g(s)]\,ds$$

Show that the critical numbers of C occur at the numbers t where $C(t) = f(t) + g(t)$.

(b) Suppose that

$$f(t) = \begin{cases} \dfrac{V}{15} - \dfrac{V}{450}t & \text{if } 0 < t \leqslant 30 \\ 0 & \text{if } t > 30 \end{cases}$$

and $$g(t) = \frac{Vt^2}{12{,}900} \qquad t > 0$$

Determine the length of time T for the total depreciation $D(t) = \int_0^t f(s)\,ds$ to equal the initial value V.

(c) Determine the absolute minimum of C on $(0, T]$.

(d) Sketch the graphs of C and $f + g$ in the same coordinate system, and verify the result in part (a) in this case.

29. A manufacturing company owns a major piece of equipment that depreciates at the (continuous) rate $f = f(t)$, where t is the time measured in months since its last overhaul. Because a fixed cost A is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

(a) Explain why $\int_0^t f(s)\,ds$ represents the loss in value of the machine over the period of time t since the last overhaul.

(b) Let $C = C(t)$ be given by

$$C(t) = \frac{1}{t}\left[A + \int_0^t f(s)\,ds\right]$$

What does C represent and why would the company want to minimize C?

(c) Show that C has a minimum value at the numbers $t = T$ where $C(T) = f(T)$.

WRITING PROJECT

Newton, Leibniz, and the Invention of Calculus

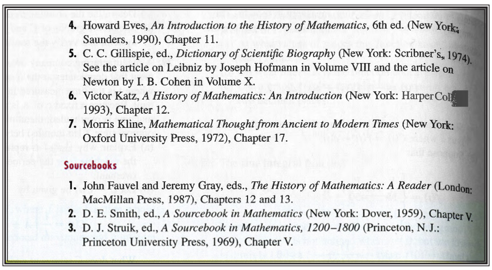
We sometimes read that the inventors of calculus were Sir Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716). But we know that the basic ideas behind integration were investigated 2500 years ago by ancient Greeks such as Eudoxus and Archimedes, and methods for finding tangents were pioneered by Pierre Fermat (1601–1665), Isaac Barrow (1630–1677), and others. Barrow, Newton's teacher at Cambridge, was the first to understand the inverse relationship between differentiation and integration. What Newton and Leibniz did was to use this relationship, in the form of the Fundamental Theorem of Calculus, in order to develop calculus into a systematic mathematical discipline. It is in this sense that Newton and Leibniz are credited with the invention of calculus.

Read about the contributions of these men in one or more of the given references and write a report on one of the following three topics. You can include biographical details, but the main thrust of your report should be a description, in some detail, of their methods and notations. In particular, you should consult one of the sourcebooks, which give excerpts from the original publications of Newton and Leibniz, translated from Latin to English.

- The Role of Newton in the Development of Calculus
- The Role of Leibniz in the Development of Calculus
- The Controversy between the Followers of Newton and Leibniz over Priority in the Invention of Calculus

References

1. Carl Boyer and Uta Merzbach, *A History of Mathematics* (New York: John Wiley, 1987), Chapter 19.
2. Carl Boyer, *The History of the Calculus and Its Conceptual Development* (New York: Dover, 1959), Chapter V.
3. C. H. Edwards, *The Historical Development of the Calculus* (New York: Springer-Verlag, 1979), Chapters 8 and 9.



4. Howard Eves, *An Introduction to the History of Mathematics*, 6th ed. (New York: Saunders, 1990), Chapter 11.
5. C. C. Gillispie, ed., *Dictionary of Scientific Biography* (New York: Scribner's, 1974). See the article on Leibniz by Joseph Hofmann in Volume VIII and the article on Newton by I. B. Cohen in Volume X.
6. Victor Katz, *A History of Mathematics: An Introduction* (New York: Harper Coll[illegible] 1993), Chapter 12.
7. Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), Chapter 17.

Sourcebooks

1. John Fauvel and Jeremy Gray, eds., *The History of Mathematics: A Reader* (London: MacMillan Press, 1987), Chapters 12 and 13.
2. D. E. Smith, ed., *A Sourcebook in Mathematics* (New York: Dover, 1959), Chapter V.
3. D. J. Struik, ed., *A Sourcebook in Mathematics, 1200–1800* (Princeton, N.J.: Princeton University Press, 1969), Chapter V.

Example 13
Writing project from Stewart (2005, 385-86).

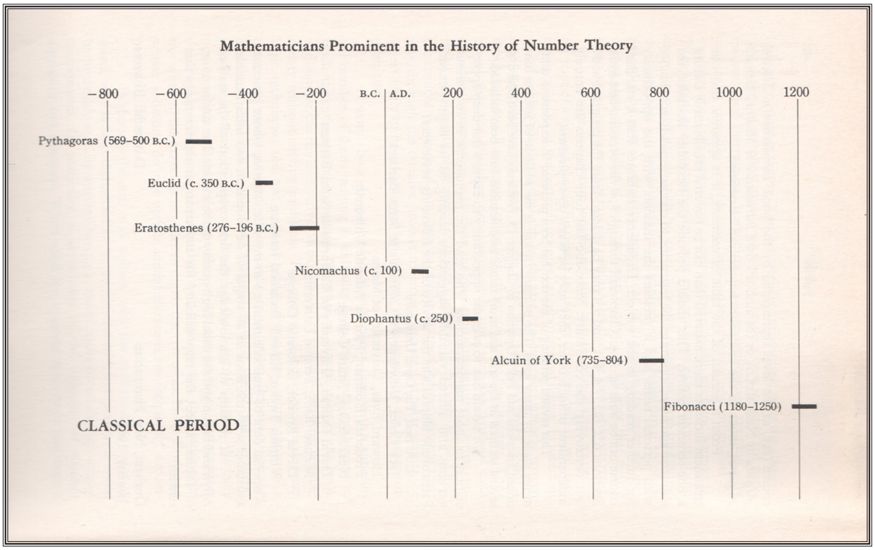
I'll turn briefly now to the biology, economics, psychology and philosophy books.[25] The biology book, *the* standard text on the subject by Campbell and Reece, runs

25 Campbell and Reece (2005), Gwartney, Stroup, Sobel, and Macpherson (2006), and Feldman, (2005).

thirteen hundred 8.5 x 11 pages in double column format, and weights 6.5 pounds. Talk about encyclopedia volumes! There's evidently *plenty* to learn in biology. There's no separate name index, no bibliography. Names are mentioned occasionally in the text, but no articles are cited. The economics book has a little more of a sense of history: numerous text boxes highlight the contributions of economists, historical and contemporary. Example 17 reproduces the thumbnail history of economics inside the front cover.

At the opposite extreme from the biology book stands the psychology book (Feldman 2005). It includes a name index of fifteen pages, at about 250 names per page, and a 46-page bibliography in triple-column format. I was so amazed by this that I got hold of a comparable book: it has a forty-two page bibliography in double-column format and an 8-pp. name index in quadruple-column format at c. 350 names per page.[26] Unlike the first book, many statements in this text are supported by citations to the bibliography, and the book begins with a brief history of psychology.

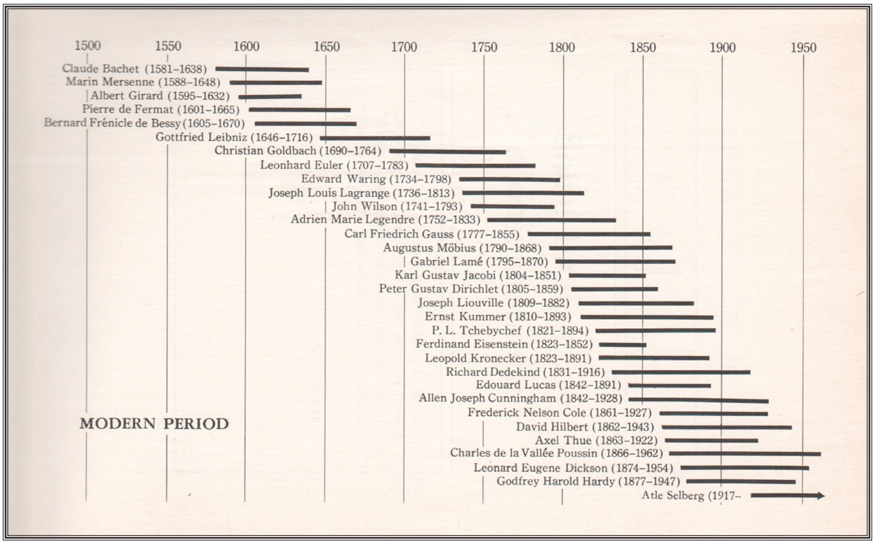
Finally, I turn to the philosophy book (most recent edition, 2013).[27] Not surprisingly, philosophers seem to have the best sense of their own history, and the



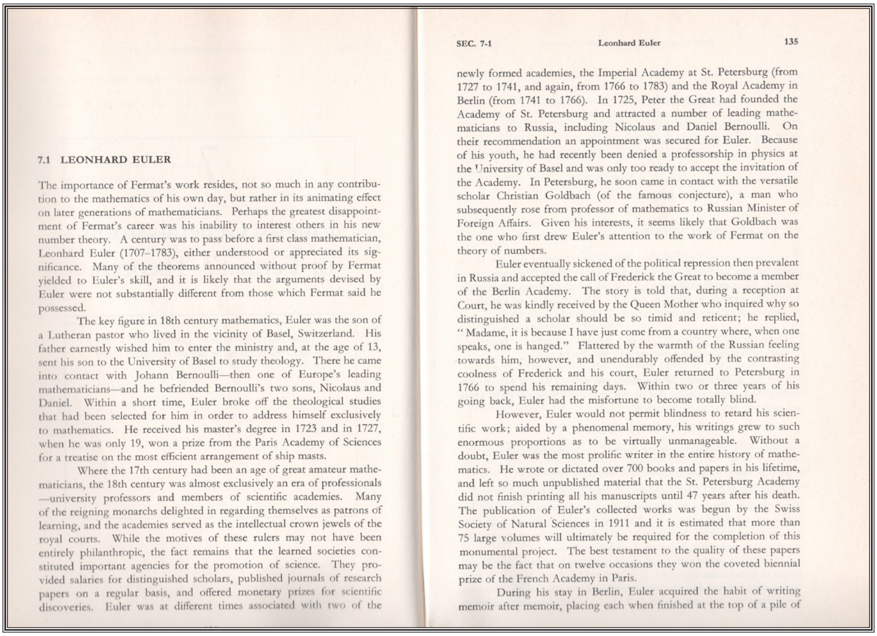
Example 14
Classical Period Number Theory Mathematicians (from Burton 1980).

26 Kalat (1996).

27 Feinberg (1978).



Example 15
Modern Period Number Theory Mathematicians (from Burton 1980).



7.1 LEONHARD EULER

The importance of Fermat's work resides, not so much in any contribution to the mathematics of his own day, but rather in its animating effect on later generations of mathematicians. Perhaps the greatest disappointment of Fermat's career was his inability to interest others in his new number theory. A century was to pass before a first class mathematician, Leonhard Euler (1707–1783), either understood or appreciated its significance. Many of the theorems announced without proof by Fermat yielded to Euler's skill, and it is likely that the arguments devised by Euler were not substantially different from those which Fermat said he possessed.

The key figure in 18th century mathematics, Euler was the son of a Lutheran pastor who lived in the vicinity of Basel, Switzerland. His father earnestly wished him to enter the ministry and, at the age of 13, sent his son to the University of Basel to study theology. There he came into contact with Johann Bernoulli—then one of Europe's leading mathematicians—and he befriended Bernoulli's two sons, Nicolaus and Daniel. Within a short time, Euler broke off the theological studies that had been selected for him in order to address himself exclusively to mathematics. He received his master's degree in 1723 and in 1727, when he was only 19, won a prize from the Paris Academy of Sciences for a treatise on the most efficient arrangement of ship masts.

Where the 17th century had been an age of great amateur mathematicians, the 18th century was almost exclusively an era of professionals—university professors and members of scientific academies. Many of the reigning monarchs delighted in regarding themselves as patrons of learning, and the academies served as the intellectual crown jewels of the royal courts. While the motives of these rulers may not have been entirely philanthropic, the fact remains that the learned societies constituted important agencies for the promotion of science. They provided salaries for distinguished scholars, published journals of research papers on a regular basis, and offered monetary prizes for scientific discoveries. Euler was at different times associated with two of the

SEC. 7-1 Leonhard Euler 135

newly formed academies, the Imperial Academy at St. Petersburg (from 1727 to 1741, and again, from 1766 to 1783) and the Royal Academy in Berlin (from 1741 to 1766). In 1725, Peter the Great had founded the Academy of St. Petersburg and attracted a number of leading mathematicians to Russia, including Nicolaus and Daniel Bernoulli. On their recommendation an appointment was secured for Euler. Because of his youth, he had recently been denied a professorship in physics at the University of Basel and was only too ready to accept the invitation of the Academy. In Petersburg, he soon came in contact with the versatile scholar Christian Goldbach (of the famous conjecture), a man who subsequently rose from professor of mathematics to Russian Minister of Foreign Affairs. Given his interests, it seems likely that Goldbach was the one who first drew Euler's attention to the work of Fermat on the theory of numbers.

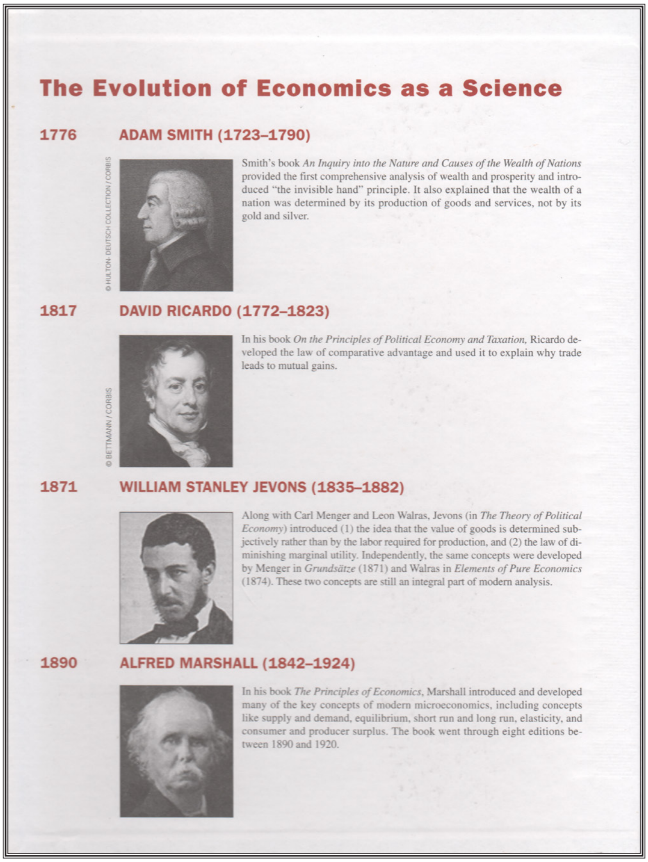
Euler eventually sickened of the political repression then prevalent in Russia and accepted the call of Frederick the Great to become a member of the Berlin Academy. The story is told that, during a reception at Court, he was kindly received by the Queen Mother who inquired why so distinguished a scholar should be so timid and reticent; he replied, "Madame, it is because I have just come from a country where, when one speaks, one is hanged." Flattered by the warmth of the Russian feeling towards him, however, and unendurably offended by the contrasting coolness of Frederick and his court, Euler returned to Petersburg in 1766 to spend his remaining days. Within two or three years of his going back, Euler had the misfortune to become totally blind.

However, Euler would not permit blindness to retard his scientific work; aided by a phenomenal memory, his writings grew to such enormous proportions as to be virtually unmanageable. Without a doubt, Euler was the most prolific writer in the entire history of mathematics. He wrote or dictated over 700 books and papers in his lifetime, and left so much unpublished material that the St. Petersburg Academy did not finish printing all his manuscripts until 47 years after his death. The publication of Euler's collected works was begun by the Swiss Society of Natural Sciences in 1911 and it is estimated that more than 75 large volumes will ultimately be required for the completion of this monumental project. The best testament to the quality of these papers may be the fact that on twelve occasions they won the coveted biennial prize of the French Academy in Paris.

During his stay in Berlin, Euler acquired the habit of writing memoir after memoir, placing each when finished at the top of a pile of

Example 16
Leonhard Euler (in Burton 1980, 134-135).

book I examined, *Reason and Responsibility*, had an excellent mixture of a historical and topical approach. Though apparently a book of "source readings," the large sections are topical, their internal organization chronological—highly suggestive for the music theorist. Thus the six parts of the book move smoothly from metaphysics to moral philosophy, carrying the titles, The Existence and Nature of God; Human Knowledge: Its Grounds and Limits; The Mind-Body Problem; Determinism and Free Will; Responsibility and Punishment; Self-Love and the Claims of Morality. The readings are from historical sources and contemporary interpretive essays, some commissioned especially for the book, which is a veritable model for an undergraduate history-of-theory course book, presuming we can frame comparable questions that suggest the larger categories. But that proved difficult to do in the Cambridge project.



The Evolution of Economics as a Science

1776 ADAM SMITH (1723–1790)

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Smith's book *An Inquiry into the Nature and Causes of the Wealth of Nations* provided the first comprehensive analysis of wealth and prosperity and introduced "the invisible hand" principle. It also explained that the wealth of a nation was determined by its production of goods and services, not by its gold and silver.

1817 DAVID RICARDO (1772–1823)

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In his book *On the Principles of Political Economy and Taxation,* Ricardo developed the law of comparative advantage and used it to explain why trade leads to mutual gains.

1871 WILLIAM STANLEY JEVONS (1835–1882)

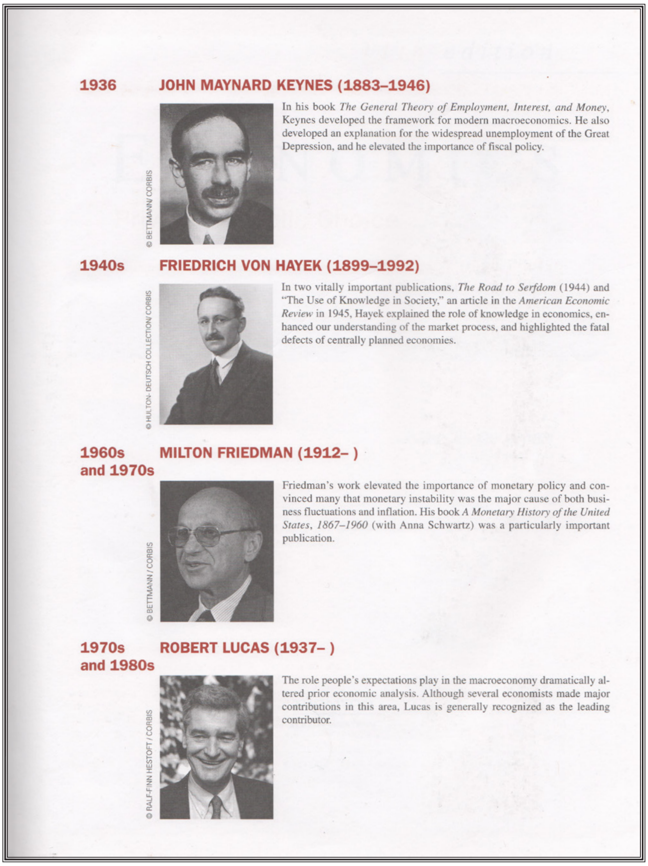
Along with Carl Menger and Leon Walras, Jevons (in *The Theory of Political Economy*) introduced (1) the idea that the value of goods is determined subjectively rather than by the labor required for production, and (2) the law of diminishing marginal utility. Independently, the same concepts were developed by Menger in *Grundsätze* (1871) and Walras in *Elements of Pure Economics* (1874). These two concepts are still an integral part of modern analysis.

1890 ALFRED MARSHALL (1842–1924)

In his book *The Principles of Economics*, Marshall introduced and developed many of the key concepts of modern microeconomics, including concepts like supply and demand, equilibrium, short run and long run, elasticity, and consumer and producer surplus. The book went through eight editions between 1890 and 1920.

Example 17

Prominent Economists, from Gwartney et al. (2006).



1936 JOHN MAYNARD KEYNES (1883–1946)

In his book *The General Theory of Employment, Interest, and Money*, Keynes developed the framework for modern macroeconomics. He also developed an explanation for the widespread unemployment of the Great Depression, and he elevated the importance of fiscal policy.

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1940s FRIEDRICH VON HAYEK (1899–1992)

In two vitally important publications, *The Road to Serfdom* (1944) and "The Use of Knowledge in Society," an article in the *American Economic Review* in 1945, Hayek explained the role of knowledge in economics, enhanced our understanding of the market process, and highlighted the fatal defects of centrally planned economies.

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1960s and 1970s MILTON FRIEDMAN (1912–)

Friedman's work elevated the importance of monetary policy and convinced many that monetary instability was the major cause of both business fluctuations and inflation. His book *A Monetary History of the United States, 1867–1960* (with Anna Schwartz) was a particularly important publication.

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1970s and 1980s ROBERT LUCAS (1937–)

The role people's expectations play in the macroeconomy dramatically altered prior economic analysis. Although several economists made major contributions in this area, Lucas is generally recognized as the leading contributor.

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Example 17 (cont'd)
Prominent Economists, from Gwartney et al. (2006).

I conclude from this look at textbooks in other disciplines that we should throw our lot in with mathematics and philosophy, the two disciplines with a historical record comparable to ours—indeed, our history is mixed with theirs! At the same time, if I thought the psychologists seemed to make a fetish of citing the present, that's not a bad thing either—in moderation.

III.

In the fall of 1999 and 2000, Matthew Brown and I co-taught a first-semester freshman theory core course at Eastman. An old department chair I worked for in the early 1970s used to say that a course is barely civilized the first time through.

We got past the nascent civility phase in our two shots at it, but the course remained experimental: there were still bugs to be worked out; but we had our successes, and some ardent fans of our approach. Our goal was to focus on music theory and its history, and related compositional assignments. Most important, we wanted to instill the theoretical/analytical approach to music in our students from the first day on. We took the "historical" and "comprehensive" from Comprehensive Musicianship, but not the neutral, formulaic chronology. Instead, we developed it from the theorist's point of view: we focused on particular theoretical topics we found essential, dealing briefly with the personalities associated with them, repertoires in chronological sequence that we knew would exemplify those topics for analysis, and compositional assignments that would emulate those repertoires. Since we worked largely before the period of the Great Composers—with exceptions, of course: we did wind up with Bach, after all!—we were able to accent the theorists that much more easily. We used 18th-century instructional manuals during the second and third units.

Example 18 shows the first page of the syllabus for the course from 2000. The three main units were: 1. Fundamentals and chant; 2. modal counterpoint and Renaissance polyphony, and 3. the figured bass and basics of the Bach chorale. Thus, by Unit III, we entered the Bach Chorale tradition of Donald Tweedy, the first theory teacher at Eastman,[28] and McHose, though with a vastly different run-up to it. The table of contents of the anthology we assembled for the first eight weeks is shown in Example 19. The other sources are well-known. Aldwell/Schachter was there principally to provide continuity into the next term, in which it was the course text; species counterpoint and figured bass were the basis of our chorale pedagogy. Most of our bass realizations and chorale composition came from phrase models we had presented and practiced all the way from the beginning of the course: to the knowledge of "chorale tunes" learned in Unit I on chant, we added counterpoint according to Fux in Unit II, and figured bass realization according to Bach, Handel and others in Unit III.

In our "comprehensive" course, we also controlled the curriculum for aural skills and keyboard audits, the former beginning in Unit I, the latter beginning with unit III on figured bass and the chorale, in week 9. We used the anthology and Fux's *Sing-Fundament* for sight-singing, and the *Sing-Fundament* (F), a few things from Salzer/Schachter *Counterpoint in Composition* (SS), and Konrad Max Kunz, Op. 14, "Two-Part Canons," for keyboard (K); the keyboard syllabus from 1999 is shown as Example 20. We got as far as elementary figured bass realization, and filling out one inner

28 Lenti (2004, 184-5).

part (the alto) of a couple of Bach/Schemelli chorales with basses; we did not teach Roman numeral progressions in keyboard or in composition, though we made a point of practicing cadence formulas throughout the course, vocally, in writing and at the keyboard.

Occasional quizzes, and exams (one on each unit) tested the historical and theoretical material. Example 21 shows our theorists' biographies, all of whom (and more) we worked into lectures, though the accent was on theories in action to generate musical materials, and to use in analysis. Study for the tests and the compositional assignments (often requiring an accompanying analysis) constituted the homework.

I'll speak briefly now about the three units. Among the more difficult topic areas to motivate in a freshman theory course are theory fundamentals. We tried to motivate them historically and theoretically from the repertoire: thus, we did as many fundamental techniques as were necessary for the music at hand in Unit I, Gregorian Chant: clefs, note reading, intervals, and theory of modality. We left rhythm for Unit II, counterpoint; its introduction with species made perfect sense, and three-part counterpoint proved to be an excellent opportunity to talk about triads. We left seventh chords, scales and other tonal theory until Unit III, chorale style.

In the first lecture I warned everyone to get the piano and their preconceived notions of the division of pitch space out of their mind, and talked about the Ancient Greeks' discovery of the perfect consonances, and their reliance on these "signposts" in an otherwise uncharted pitch continuum. From there we studied their derivation of the whole step from the difference of a fourth and fifth, and the "remainder" of two whole steps from a fourth as their half step—the filling-in between the signposts, and the concatenation of these tetrachords into a 2-octave range of variable "steps" in the different genera. So that the students could get a conceptual grasp on the interval-sizes signified by ratios, I taught them basic tuning arithmetic, the Ellis 1200-cent-per-octave system, and logarithmic conversion between them (not all that tough for these kids straight out of high school, armed at that time mainly with pocket calculators, but today with phones with scientific calculator apps). I quickly turned to the diatonic genus only, but the students had at least some idea of the etymology of diatonic, chromatic and enharmonic, and the profound change in their meanings as we use them now. As the material developed, in lieu of a textbook, I distributed a detailed "Study Guide" summary to assist in test preparation.

By week three, I explored the diatonic materials in some depth in an eclectic theory + history-of-theory handout shown as Example 22. Matthew responded with a lecture on Guido's "Ut queant laxis," in which he starts with a pitch and interval

TH 101: Freshman Theory, First Term
Course Syllabus for Fall 2000

Brown/Wason and Colleagues

Course Description: The course is divided into three units: 1. introduction to fundamental aspects of pitch and rhythm, and to melody, motive, theme, cadence and phrase; 2. the basics of 2- and 3-part modal counterpoint; 3. fundamentals of figured-bass and early 18th-century chorale-style.

Required Materials: Brown/Wason, *Anthology of Gregorian Chant, 16th-Century Counterpoint, and Bach/Handel on Figured Bass*
Johann-Joseph Fux, *Gradus ad Parnassum*, Alfred Mann, tr. (NY: Norton, 1965 and many later editions)
J.S. Bach *371 Chorales and 69 Chorale-Basses* (NY: G. Schirmer)
Aldwell/Schachter, *Harmony and Voice Leading* Harcourt, Brace; 2nd ed. 1989.

Syllabus

I. Fundamental Aspects of Pitch and Rhythm; Introduction to Melody, Motive, Cadence and Phrase.

wk. 1	A.	The Notion of "Interval" and the discovery of "consonance"
	B.	Notation as a "theory" of music
wk. 2	A.	Introduction to Melody
	B.	Melodic Prototypes; melody composition
wk. 3	A.	Theory of Mode and Scale
	B.	Melodic Analysis

II. Modal Counterpoint

wk. 4	A.	Mensuration and Rhythm in one part; pulse, accent, meter
	B.	species rhythm; levels of rhythm and consonance/dissonance
wk. 5	A.	The Origins of Polyphony
	B.	Theory of Counterpoint in 2 parts
wk. 6	A.	Filling in consonant harmonic space (2nd Species)
	B.	Delaying the progress of melodic motion (4th Species)
wk. 7	A.	The origins of polyphonic melody (3rd Species)
	B.	Putting it all together in two-part composition (5th Species)
wk. 8	A.	History of Counterpoint in 3 and more parts.
	B.	Theory of Counterpoint in 3 and more parts.

Example 18
Brown and Wason TH101 Syllabus (2000).

Table of Contents

Example 19
Brown and Wason Anthology Table of Contents.

TH 101/Fall '99
Brown/Wason

Schedule of Assignments for Keyboard Audits

The Keyboard Audit book contains more material than you will cover in the audits, but then there will also be many different levels of experience and ability; we want to have something available to challenge all students. The following assignment list may be too easy for some students, too hard for others; in that event, your individual keyboard auditor may adjust it accordingly. The important thing is that *all* students make continuous progress from where they start.

The audit book consists of musical examples from Fux *Fundamentals of Singing*, Salzer/Schachter *Counterpoint in Composition* , and Kunz *200 Short Canons for the Pianoforte*. In the following schedule, the sources are referred to as F, SS, and K, followed by an example number.

- In the one-part Fux exercises, first be able to sing the exercise. Then play it with *each* hand, separately.
- In *all* of the two-part pieces, start by learning to sing one part at a time. Then learn to play one part at a time. In deciding on fingering, remember that your thumb, pointer and middle finger are the strongest: you'll want to favor these; avoid straining your weaker fingers.
- After you've learned to play the parts, start learning to sing (in an octave that is comfortable) one part while playing the other one; then reverse the sing/play relationship. At this point begin work on playing the two parts together. This an *ear-training exercise* in hearing two-part counterpoint.
- In the last three weeks, we will work on two chorales from the Bach/Schemelli chorales that you have already purchased. Work through each chorale, phrase by phrase; in each, practice singing the chorale tune while playing the bass line. Only when you are able to do that should you practice playing the tune and the bass. After you have learned the soprano and bass, add a *single* inner part in the right hand (an alto, below the tune), consisting of notes prescribed by the bass figures.

The following schedule gives the Monday of the week each assignment is **due**:

week	
wk. 9 (Oct. 25)	F(one part) 1-12; SS 1-49, a-c; K 1-2
wk.10 (Nov. 1)	F(one part) 14-22; SS 2-26, a-d; K 5-6
wk.11 (Nov. 8)	F(one part) 23-33; SS 4-16, a-d; K 7-8
wk. 12 (Nov. 15)	F(one part) 34-43; SS 3-41, a-d; K 10-11
wk. 13 (Nov. 22)	Bach/Schemelli 60 (1st six measures); SS 5-21, e-h; K 14-15;
wk.14 (Nov. 29)	Bach/Schemelli 60 (finish); K 14-15
wk. 15 (Dec.6)	Bach/Schemelli 50; F (Duette) 1
wk. 16 (Dec. 14-17)	**Final Examination**

Example 20
Brown and Wason, TH101 Keyboard Assignments (1999).

TH 101: Monuments in the History of Music Theory (6th Century BCE—18th Century CE)

Pythagoras: 6th century BCE; allegedly, the sound of hammers in a blacksmith shop (tuned by chance an octave apart) led P. to experiment with various sounding objects, and discover the ratios of the perfect consonances: perfect octave (1:2), perfect fifth (2:3) and perfect fourth (3:4). This is the origin of the notion of interval as a ratio of two quantities. The doctrine of "harmony of the spheres" is associated with P. and passed on by Plato and others.

Aristoxenus: 4th century BCE; student of Aristotle (and thought he would be his teacher's successor as head of the Lyceum, but he got cut out of the job); his *Harmonics* is the first surviving treatise on music (though not complete, much of it remains; his *Rhythmics* is more fragmentary). In opposition to the Pythagoreans, he originated the idea that an interval is a distance between two pitches.

Boethius: late 5th, early ~ Century CE; Consul to Emperor Theodoric in the final stages of the Roman Empire (executed by Theodoric, c. 542); wrote treatises on mathematics, logic, philosophy, and music; transmitted Greek learning (particularly of the "quadrivium" = arithmetic, music, geometry, astronomy) to musicians and scholars in the Middle Ages.

Hucbald: 9th century (d. 930); monk of the Abby of St. Amand; reinterpreted the Greek tetrachordal "Greater Perfect System" for use in the Middle Ages; prototypical tetrachord became "tetrachord of finals" DEFG (TST), instead of the Greek EFGA (STT); one of the first descriptions of the Medieval Modal System (the "8 Church Modes")

Guido: l l" Century; monk whose fame assumed legendary proportions; credited with the invention of: 1. Staff notation; 2. Hexachordal solfege system (from "Ut queant laxis"). Discusses modes by final and range; *Micrologus* is one of the first treatises on composition.

Prosdocimus: late 14th early 15th-century composer and professor at University of Padua; codified the principles of strict counterpoint (note against note) in 2 voices in his *Contrapunctus (1412).*

Tinctoris: Flemish 15th-century composer and author of an important treatise on counterpoint (1477); expands principles of counterpoint to cover "florid counterpoint," becoming the first to describe the use of dissonant intervals in counterpoint; distinction between written *(res facta)* and "improvised" *(supra librum)* counterpoint; voice intervals

Example 21

Theorist Biographies, from Brown and Wason TH101 curriculum.

figured with reference to the tenor; "Forget about music composed more than 40 years earlier!"

8. **Glarean**: Swiss 16th-century humanist scholar who revived Ancient Greek music theory; author of the system of 12 modes which he thought filled out the previous Medieval (8-mode) system the way it should have been all along (divide all species of the 8ve into 4th+5th and 5th+4th; cut the two 8ves that yield tritones, and you get 12).

9. **Zarlino**: 16th century composer and author of the most renowned treatise on Renaissance counterpoint; the second edition (1573) also passed on Glarean's version of the modes; voice intervals figured from the bass; recommends the use of triads.

10. **Fux**: Austrian 17th, early 18th century composer, Master of Music at St. Stephan's in Vienna, and author of *Gradus ad Parnassum* (1725), which canonized five "species" of counterpoint (1st = strict counterpoint, while 5th = florid). Mozart, Haydn, Beethoven and Schubert all studied this book.

11. **Campion**; late 17th, early 18th century author of a figured bass manual that was the first to make a fully developed method of the "Rule of the Octave" scheme for teaching keyboard players to harmonize an unfigured bass. (*Treatise on Accompaniment and Composition, according to the Rule of Octaves* [Paris: 1716])

12. **Heinichen**: 18th-century German composer and author of the most comprehensive figured-bass manual of the century (*Der Generalbass in der Composition*; Dresden, 1728); listed the figures of thirty-two "chords of thoroughbass" in numerical order; epitomizes the "Bach Style" of Northern Germany.

13. **Rameau**; the most famous French composer of the first half of the 18th century, and author of *Traité d l'harmonie* (Paris: 1722), a revolutionary work. R. developed his notion of a "fundamental bass" (*basse fondamentale*), according to which **every** "chord" had a fundamental bass that was imagined under its real sounding bass. The doctrine of **harmonic inversion** was implicit: **any** note of the chord might become the sounding bass, but the chord's identity would remain the same. Using the fundamental bass and harmonic inversion, R. was able to claim that **all** of the chords of figured-bass practice were derived from 5/3 and 7 chords. Using the fundamental bass to articulate "rules" of harmonic progression, he was the first to place various chords in equivalent functional categories.

Example 21 (cont'd)

Theorist Biographies, from Brown and Wason TH101 curriculum.

listing, according to "chunks" demarcated by cadence points (Example 23, fig. 2-3), and winds up with his own pitch-hierarchic analysis of the chant (Example 23, fig. 4), presented as a compositional method. Very ingenious! It's long been surmised that Guido composed this chant, or altered an existing one, to demonstrate the hexachord. I like to think Guido Monaco—Guido the monk—as he is remembered today in Arezzo, would have blessed this analysis (Examples 24 and 25).

The composition of chants with analyses finished Unit I, and we were off to Unit II on counterpoint. Matthew essentially took over Unit II, since he has long made a detailed study of Fux and the species. Example 26 reproduces his method for composing a first-species counterpoint. The unit included reading and discussion of Fux as we went along.

TH 101: Lecture 3

The Gamut Generated by Fifths; Species of the Consonances; Mode

Figure 1: The Gamut (Gamma-Ut) Generated by Fifths: a theoretical explanation that may have historical significance--and certainly has practical significance.

Collection-Type	m2/M7	M2/m7	m3/M6	M3/m6	P4/5	TT
1. 4th/5th					1	
2. "4th chord"		1			2	
3. "I Got Rhythm"		2	1		3	
4. Pentatonic Scale		3	2	1	4	
5. Guido's Hexachord	1	4	3	2	5	
6. Diatonic Scale	2	5	4	3	6	1

Figure 2a: Ancient Greek (descending)

[A B C D E F G a b c d e f g a]

T S T T S T T T S T T S T T

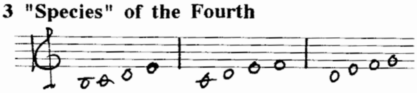
2b:

The Medieval Gamut Γ (with Gamma-Ut)

[A B C D E F G a b c d e f g a]

(T) T S T T S T T T S T T S T T

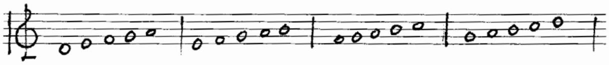
Figure 3: 3 "Species" of the Fourth



"Greek" STT (B/E) TTS (C/F) TST(D/G)

Medieval TST STT TTS

Figure 4: 4 Species of the Fifth (in Medieval Ordering)



TSTT (D/A) STTT (E/B) TTTS (F/C) TTST (G/D)

Example 22

Wason TH101 Lecture: History of Theory.

Figure 5: **a. 7 Species of the Octave** **b. as $5^{th}+4^{th}$, $4^{th}+5^{th}$**

TSTTTST (D/D)
STTTSTT (E/E)
TTTSTTS (F/F)
TTSTTST (G/G)

TSTTSTT (A/A)
STTSTTT (B/B)
TTSTTTS (C/C)

TSTT	**TST**
STTT	**STT**
TTTS	**TTS**
TTST	**TST**
TST	**TSTT**
STT	**STTT**
TTS	**TTTS**
(TST	**TTST)?**

Figure 6: **<u>The 8 Medieval Modes</u> (c. 1000AD): range (*ambitus*) is shown in solid note heads; final (*finalis*) shown with a whole note: the odd numbered modes are <u>authentic</u>, the even ones are <u>plagal</u>. Eventually, the Greek ethnic names were associated with them as follows: 1: dorian; 2: hypo-dorian; 3: phrygian; 4: hypo-phrygian; 5: lydian; 6: hypo-lydian; 7: mixo-lydian; 8: hypo-mixo-lydian.**

Figure 7: **<u>The 12 Renaissance Modes</u> (c. 1550AD): Glarean's "completion" of the Octave Species, Analyzed as Species of the $4^{th}+5^{th}$, or $5^{th}+4^{th}$.**

TTSTTTS (C/C) (<u>Ionian Mode</u>)	**TTST TTS**	**4/3**
TSTTTST (D/D)	TSTT TST	1/1
STTTSTT (E/E)	STTT STT	2/2
TTTSTTS (F/F)	TTTS TTS	3/3
TTSTTST (G/G)	TTST TST	4/1
TSTTSTT (A/A) (<u>Aeolian Mode</u>)	**TSTT STT**	**1/2**
TTSTTST(G/G)	**TTS TTST**	**3/4**
TSTTSTT (A/A)	TST TSTT	1/1
STTSTTT (B/B)	STT STTT	2/2
TTSTTTS (C/C)	TTS TTTS	3/3
TSTTTST (D/D)	TST TTST	1/4
STTTSTT (E/E)	**STT TSTT**	**2/1**

Example 22 (cont'd)
Wason TH101 Lecture: History of Theory.

Figure 1

Hymn to St. John the Baptist

That thy servants may freely sing forth the wonders of thy deeds, remove all stain of guilt from their unclean lips, O Saint John.

Figure 2

	C	D	E	F	G	A	Total
Chunk 1	1	3	1	1	-	-	6
Chunk 2	1	3	2	-	-	-	6
Chunk 3	1	2	3	1	1	-	8
Chunk 4	-	2	-	2	2	1	7
Chunk 5	-	1	1	2	3	1	8
Chunk 6	-	-	-	1	2	4	7
Chunk 7	1	2	2	1	1	-	7
Total	4	13	9	8	9	6	49

Figure 3

	U	m2	M2	m3	M3	P4	P5	Leaps
Chunk 1	-	-	3	2	-	-	-	2
Chunk 2	3	-	3	-	-	-	-	-
Chunk 3	1	1	4	1	1	-	-	2
Chunk 4	1	-	4	2	-	-	-	2
Chunk 5	-	2	4	-	-	2	-	2
Chunk 6	1	-	4	-	1	-	1	2
Chunk 7	-	1	5	-	1	-	-	1
Total	6	4	27	5	3	2	1	11

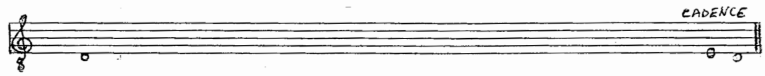
Example 23
Brown TH101 Lecture: Guido and Solmization.

Figure 4

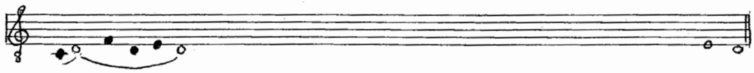
Procedure for composing a plainchant

1. Find the final near the beginning and at end of the melody.

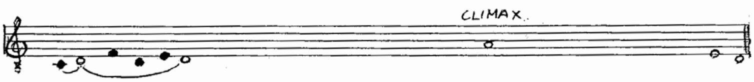
2. Reinforce the final at the end by a cadential descent of a whole step.



3. Establish the final at the start by elaborating it with the leading tone and the third above. These pitches give the melody a major or a minor feel.



4. Find a climax not more than an octave above the final and about two-thirds the way through the melody.



5. Join the beginning to the climax and the climax to the end. Make a general sketch, then fill it out in more detail. Make sure that the melody mainly moves by step and obeys the laws governing melodic leaps. Be careful that the climax is properly prepared and doesn't jump out of nowhere.

6. Check to see that you melody sounds good and has an interesting shape; it should resemble a series of waves that gently rise and fall.

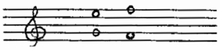
Example 23 (cont'd)
Brown TH101 Lecture: Guido and Solmization.

Example 24
Via Guido Monaco, in Arezzo, Italy.

Example 25
Statue of Guido (with pigeon on his head) in Arezzo, Italy.

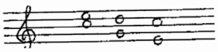
Figure 3

Concepts:
Monophony is a texture that consists of a single melody.
Homophony is a texture that has several simultaneous melodies that move in the same rhythm.
Polyphony is a texture that has several simultaneous melodies that move in different rhythms.
Contrary motion is a type of motion in which two melodies move in opposite directions.



Oblique motion is a type of motion in which one melody remains fixed while and the other(s) move up or down.

Similar motion is a type of motion in which two melodies move in the same direction by different intervals.



Parallel motion is a type of motion in which two melodies move in the same direction by the same interval.

Melodic Motion and Closure

a. If the counterpoint is maximally closed, then it begins on ^8/U or ^5, and ends by step up
b. If a counterpoint is prototypical, then it essentially moves by whole– and half step, with no repeated tones.
c. If leaps occur, then they are never larger than an octave and never encompass diminished/augmented intervals or the interval of a seventh.
d. If leaps occur, then they never appear successively in the same direction and are normally approached/departed by step in the opposite direction.

Relative Motion and Closure

a. If a counterpoint is prototypical, then it essentially moves in contrary motion with the *cantus firmus*.
b. If a counterpoint and the *cantus firmus* move in the same direction, then parallel perfect octaves and fifths do not occur between successive notes.
c. If a counterpoint and *cantus firmus* move in the same direction, then they can never contain more than four successive parallel thirds/sixths.

Vertical Disposition

a. If the counterpoint is prototypical, then it begins on a perfect consonance and ends on a unison or octave.
b. If the counterpoint is in First Species, then is always forms consonant intervals with the *cantus firmus*
c. If the counterpoint is prototypical, then unisons only occur at the beginning or the end.

Example 26
Brown on composing first-species counterpoint.

Figure 4

Procedure for composing a First Species counterpoint

==

1. Find a perfect consonance at the beginning and the final at end of the melody.

2. Reinforce the final at the end by a cadential ascent by step.

3. Find a climax (i.e. single highest note) for the counterpoint; avoid putting it in the same place as the climax of the *cantus firmus*.

4. Join the beginning to the climax and the climax to the end. Make a general sketch, then fill it out in more detail. The counterpoint should mainly move by step and obey the laws governing melodic leaps, it should only use consonant intervals with the *cantus firmus* (8, U, 5, 3, 6) and should not contain parallel perfect octaves and fifths between successive notes. Only use unisons at the beginning or the end and do not repeat notes immediately.

5. Check to see that the counterpoint sounds good and has an interesting shape; it should resemble a series of waves that gently rise and fall in contrary motion with the *cantus firmus*. Get in the habit of indicating the vertical interval between the two voices; it makes it easy to spot stray dissonances and parallel perfect octaves/fifths.

==

Example 26 (cont'd)

Brown on composing first-species counterpoint.

In the interests of time I move very briefly to Unit III, in which Matthew and I played more or less equal roles. I started by laying out two practical problems of eighteenth-century keyboard pedagogy: how to order and teach the plethora of chords in the figured bass, which could now include virtually any interval on downbeats, and how to accompany an unfigured bass. Example 27 reproduces the handout that summarized some points from my introductory lecture. From there on, our pedagogy was not all that unusual, at least by eighteenth-century standards: we practiced figured bass realization (I avoided the term harmonization), Bach/Schemelli soprano/bass chorale completion of first an alto, and then two voices, and finally composing a chorale from a soprano cantus firmus. With preparation in counterpoint, six weeks proved to be enough for at least a solid introduction to chorale composition.

IV.

Now to the peroration and my big questions: 1. why has so little of the history of theory filtered down to undergrad texts; and 2. what's to be gained by integrating some of it into them?

I. As to the first question, I think there are at least two answers: first and foremost, there's the music itself—the reason most of us got into music theory: I wanted to know how it worked and how I could do it better, but the "how" remained secondary to the music. The beauty of mathematics resides within mathematics, that of philosophy in age-old questions that continue to fascinate. But we must deal with the constant back-and-forth between the aesthetic artifacts we hold dearly and want to elucidate, if not "explain," and the means by which we do so, which, in the best examples of it, has its own elegance and history. Going too far in the latter direction may seem to take time away from the music, and to risk alienating those students who naively find music theory irrelevant to the "mysteries" of music. We tend to see many of these alleged mysteries instead as "puzzles." (I think here of Richard Feynman's distinction between the two).[29] There are surely mysteries as well, but not all is "mysterious."

Second, though the history of those thinking about music-technical problems is a very long one, that of the American Academic Music Theorist is very short. Before the

29 Mysteries are questions we can only ask and ponder, but never answer; "puzzles" are questions that on first blush may seem just as impenetrable, but with work and thought—very likely with the work and thought of many—may ultimately be answered. Needless to say, Feynman concentrated on the puzzles. The source is an interview with Feynman on PBS, the date of which I cannot remember.

TH 101

Allowable Chords and Chord Progressions in the Common-Practice Style

A. <u>Possible Chord "Types" vs. Possible Chord "Progressions."</u> Beginning in the 17th century, the important status of the bass voice means that we reckon chords from the bass up. By "type," we refer to the kinds of chords (combinations of intervals) that are possible in "structural" positions (generally on the beat, not between). Major and minor triads in 5/3 and 6/3 "inversions" are admissible (though their inversional relationship was not always recognized); diminished triads in 6/3 are fine, but infrequent in 5/3; 6/4s are possible only if treated specially (the 4th from the bass must be treated as some form of dissonance).

The *seconda prattica* ("second practice") of the mid- to late sixteenth century ushered in a number of profound changes in compositional technique. Among these was the use of dissonant intervals *beyond the level of sub-metric embellishment*. When a dissonant interval lasts for the same duration as other supporting intervals, we call the dissonant interval a **chordal dissonance.** The use of chordal dissonance greatly expands the possible chord-types that may occur. The phenomenon of chordal dissonance starts with the 6/5, but the 7th-chord and its "inversions" become increasingly viable: this means that 6/5, 4/3, 4/2 and 7 are possible. In fact, the idea that a 7th (or its inversion, the second) can be a (relatively consonant) "chord tone," means that **any interval** can achieve *relatively* consonant status. How do we classify all of these new "chords"?

Chord "progression" refers to the motion of one chord to the next. Are there "rules," *beyond* the rules of voice-leading we have already learned? Or can any chord follow any other, as long as the voice leading is correct?

B. <u>The Inductive Approach to these Problems: the "Figured Bass Treatise:"</u> Figure 1 shows one attempt at organizing the possible chord-types of thorough-bass practice. Heinichen, the author of this example, wrote the most comprehensive "figured-bass treatise" (Dresden: 1728). Explain the logic of Heinichen's classificational system, which is an attempt to define and organize "chord types."

Figure 1

	2	3			4					5		6					
Gewöhnl. Signaturen des General-Basses	2	3 ♮	♯	♭	4 2	4♯	4 3	4	4 3	5♭	5+	6	6 4	6 5 4 3	6 6♭ 4♯	6 5	6 5 4
Die dazu gehörige Stimmen.	6	5	5	5	6	6	6	5	5	6	3	3	8	8	2	3	8
	4		3	6		2		8	8	3 *8	8	*8				*8	

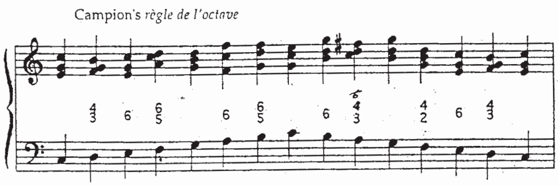
	7								9						
Gewöhnl. Signaturen des General-Basses	7	7 6	7 2	7 4 2	7 4	7 6 5 6	6 7 5 6	6 7 6 5	9	9 8	9 4	9 6	9 7	9 7 4	9 8 7 6 5 6
Die dazu gehörige Stimmen.	5	3	4	*5	5	3	3	3	5	5	5	3	3	5	3
	3 *8	5	*5		*8	*8	*8	*8	3	3			3		

Figure 2 shows an attempt to deal with the second problem: chord "progression." Campion (Paris: 1716) presents one form of a paradigm of chord-choice known as the "rule of the octave" (*Règle de l'Octave*); this is an attempt to answer the practical problem: suppose the keyboard player has to improvise the correct chords to accompany a bass.

Example 27

Wason's Introduction to figured bass.

Figure 2



Campion's example demonstrates the following two paradigmatic progressions (among others):

1. If the bass ascends through scale-degrees 1, 2, 3 (with 1 and 3 harmonized by forms of the tonic chord), then 2 may not be harmonized by II. Campion's 4/3 is typical, but 6 and even 6/4 are possible "passing chords."

2. When the bass ascends 5, 6, 7 (with 5 and 7 harmonized by forms of the dominant chord), the figure 6 creates an acceptable "passing chord" on scale-degree 6.

The example also demonstrates the correct usage of the dissonant figures, 5/6, 4/3, 4/2:

1. In using a 6/5, the 5 must be prepared by common tone, and the 5 must resolve down by step while the bass moves up by step. (Only the 6/5 on the dominant may be treated more freely, though Campion's 5 is prepared by common tone even here.)
2. All three instances of the passing 4/3 demonstrate precisely the same voice leading: bass moves by step in passing motion (same direction); common tones and closest way by step in other voices.
3. The bass of the 4/2 must descend by step.

C. The Deductive Approach: Jean-Philipe Rameau, *Traité de l'harmonie* (Paris: 1722)
Rameau started with his idea of the "fundamental bass" (*basse fondamentale*)--the idea that below **any** sounding chord there was an "imaginary bass" that is its "source." With regard to the problem of allowable chord-types, this means that 5/3, 6/3 and 6/4 chords could be thought of a representatives of the **same** fundamental bass. Though others had had the idea that triads were important, Rameau related **all** triads through this "theory of inversion," and went on to treat "7th chords" the same way. Thus, with regard to the problem of allowable chord-types, he maintained that all chords were derived from triads and seventh chords. (Return to Heinichen's list and see if this works.)

Rameau also made use of the same notion of the "fundamental bass" to talk about the problem of chord progression. It made sense to him that the fundamental bass would prefer to travel in the **same intervals** that made up his triads and seventh chords,: thus he preferred progressions by 5th and 3rd (figured in chord roots), and only grudgingly admitted progressions by step as "exceptions." Thus Rameau is the beginning of attempts to formulate "rules of harmonic progression;" unfortunately, three hundred subsequent years of commentary and criticism can't be summarized here without going over my 2-page limit.

Example 27 (cont'd)
Wason's Introduction to figured bass.

institutionalization of Music Theory in the American academy,[30] there were workaday teachers of aural skills, harmony and counterpoint—with lots of hours of teaching, focused on those subjects in necessary, but not very interesting, classes. From those classes arose our mandate, just as the mandate of historical musicology arose from basic "music history:" it's sobering to realize that until the early 1970s there were completely separate departments of undergraduate music history, graduate music history, and musicology at the Eastman School; and expectations and working conditions were quite different in each.

But times changed—at Eastman and elsewhere. Certain graduate programs, the driving forces behind them, and generally young and enlightened deans (some from those very same programs) found better ways to fill those teaching jobs. To keep them, we must walk a path that doesn't stray too far from our original mandate, risking our stakes there, or too far from our newer calling, risking our status in the larger world of scholarship. We *must* do better than those old-regime pedagogues in *everything* we do. But it's easy with undergraduate teaching to fall back into the way theory's been taught in the past—as dogma—the way some of us were taught.

Now to the second question: what's to be gained by including some of the history of theory in undergraduate textbooks?

First and most important, the history of theory provides a larger narrative in which to embed the technical content, connecting both to our own past and to that of other disciplines—which our students may be studying. A reviewer of what he calls "liberal" texts in physics, writes:

> [These convey] the concepts of science and additionally, a sense of the life and times of scientists, of the social circumstances that called forth the scientific developments, of the difficult birth of new scientific concepts and debate over their legitimacy. These texts give a sense of science as a part of culture, and usually there is something of a story line. Professional texts lack a story line: concepts, definitions, refinements, model problems and end-of-chapter exercises are the staple.[31]

We music theorists have only had "professional textbooks." But as we move out into the larger scholarly world, writing and *using* our history is part of that process. Like our reviewer, I'm calling for liberal music theory textbooks usable even in professional settings. To my knowledge they don't yet exist in English, but

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30 See Girard (2007).

31 Matthews (2000, 323). I should like to thank Professor Randall Curren, chair of the Philosophy Department of the University of Rochester, for drawing my attention to this book.

at least one new one does in German,[32] demonstrating that it's possible to take such an approach and still retain hard-core music theoretical/analytical content. The larger narrative even has practical benefits. The attribution of theoretical/analytical techniques provides a model of scholarship, by example. Isn't it better to learn to use the library, bibliographic conventions and the scholarly critical apparatus by example from your own textbook, instead of in a bibliography course? And attributions can also aid memory. Matthew and I used photos of busts, portraits, etc., when available: remember Guido Monaco—or Boethius? Example 28 provides a sketch of some possible attributions to start you thinking.

Second, as the author of the book on number theory wrote, "the historical remarks … bring out the point that number theory [read "music theory"] is not a dead art, but a living one… They reveal that the discipline developed bit by bit with the work of each individual contributor built upon the research of many others."[33] That's the message I wish our students got more often from their theory courses! After all, there *are* potential music theorists—both full- and part-time—in our student audience, and we want to teach them how we think; what better way than by showing music theory's richness and diversity in the past and present?

And Third, providing *no* history amounts essentially to a sort of naïve presentism—music theory as the invention of author X—or worse: that's just the way it's always been and always will be. Presentism—the "Whig History," as it's sometimes called—is a contentious issue among historians of science, but more problematic still in music theory, where fundamental epistemological questions remain unsettled. Though I'm sympathetic to the presentist argument—as I think most theorists are!—it can't be assumed: we must ask whether we have the "right" answers, or only the ones that are "right" for the field of music theory *today*—whether music theory is a "fashion industry," as one former colleague put it. We need to confront the question each time with persuasive musical and music-theoretical answers.

In closing, as one of those who works in the history of theory, even in retirement—or even more in retirement—I'll take some of the blame. Despite publication of much of our research over the last fifty years, it remains difficult to piece it all together. Even the *Cambridge History*, a heroic undertaking by its editor, Thomas Christensen, is a long read, its many specialized chapters difficult to synthesize into a whole. But until a more synoptic view of that whole arrives—and I don't see one on the

32 See, for example, Mencke (2015 and 2017).

33 Burton (1980, v-vi).

Are Attributions and Eponymous Laws Possible in Music Theory?

1. Pythagoras's Interval Ratios
2. Aristoxenus's Linear Interval
3. Hucbald's tetrachord of finals
4. Pesudo-Odo's Gamut
5. Guido's Hexachord
6. Guido's affinities
7. Garlandia's consonance/dissonance scale
8. Prodocimus's note-against-note counterpoint
9. Tinctoris's dissonances in florid counterpoint
10. Zarlino's Senario
11. Glarean's Twelve Modes
12. Mersenne's Equal Temperament
13. Rameau's Law of Chord Progression
14. Kirnberger's Non-Essential Chords
15. Sechter's "Hybrid Chords"
16. Schenker's *Stufen*
17. Babbitt's Common-Tone Theorem
18. Cone's Hypermeter
19. Forte's Nexus Set
20. Lewin's GIS

Example 28
Attribution of Eponymous Laws of Music Theory to Important Music Theorists.

horizon—you're on your own with it. Notwithstanding, the history of theory is to me, most importantly, an *attitude* towards teaching and towards writing music theory and analysis: it's wanting to find out where things came from—including what you think are your own "original" ideas; it's not being satisfied with one explanation, but wanting to restage the controversy between alternatives when possible. I hope you'll keep that attitude in mind in the future as a possible approach to teaching and writing textbooks.

Thanks very much, and let's get to the discussion!

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