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Pitch-Class Poker

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PITCH-CLASS POKER

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Pitch-Class Poker is a game that utilizes familiar analytic tools and concepts from atonal pitch-class set theory. Playing Pitch-Class Poker challenges the music theorist or composer to view these tools and concepts from an unusual perspective, leading to a different and hopefully deeper understanding of the theory. The beginning player must be able to calculate the interval vector for a collection of pitch-classes. Thus, for a student learning the basics of atonal theory the game provides an interesting way to practice finding the total interval content of a collection of pitch-classes. An experienced player will find that developing a strategy for the game requires a more subtle and sophisticated understanding of abstract inclusion relations in atonal theory. The rules of Pitch-Class Poker resemble the rules of traditional poker. The procedure for dealing, betting, and playing is identical in both games.[1] However, in Pitch-Class Poker a new method for ranking hands from high to low based on the total interval content or interval vector of the pitch-class set represented by the cards held in each hand replaces the traditional ranking scheme where four of a kind ranks higher than a full house, but lower than a straight flush.[2] Draw poker, stud poker, and other variations of the basic traditional game can be played depending on the interest of the players.

Two to nine may play, although the game is best for five to seven players. Eight or nine can play only Pitch-Class Stud Poker variations of the game.

A deck of 48 cards is required. Each card shows a specific pitch in a specific register and clef. C2 (cello C) through B2 are shown in bass clef, C3 (viola C) through B3 are shown in tenor clef, C4 (middle C) through B4 are shown in alto clef, and C5 through C6 are shown in treble clef. The four clefs replace the four suits in a traditional deck, so a normal five-card poker hand represents a pentachord chosen from a gamut of four octaves. Note, however, that when calculating the interval vector for a hand, the register and clef of each pitch is ignored and the four-octave representations of each pitch class are considered equivalent. Within each clef or suit, B is considered the high card, while C is the low card.

Alternately, a regular deck of 52 cards may be used if the four Queens are removed. Each card represents a pitch-class: King=C natural, Ace=C sharp or D flat, two=D natural, and so on until Jack=B natural or C flat. Suits might indicate the clef and register of each pitch, but need not be formally designated as such since the interval vector is calculated using pitch classes. Using a traditional deck the King, representing pitch class 0, is the low card, while the Jack, representing pitch class 11, is the high card. In the basic version of Pitch-Class Poker, called Tritone Trump, the hand with the most tritones (interval class 6 or ic6) in the interval vector ranks highest. If no hand contains a tritone, the hand with the most perfect fourths or fifths (ic5) wins the game. Similarly, if no hand contains ic5 or ic6, the hand with the most ic4's wins, and so forth. Figure 1 shows a specific case.

The hand (C#,E,F,F,B) or (A,4,5,5,J) using traditional cards has the interval vector <111111> and ranks higher than the hand (C,E,F,G,G#) or (K,4,5,7,8) with the interval vector <212320>. It is important to remember that a duplicate entry in the hand, such as the F or five in the hand above, is not included in the calculation of the interval vector, just as a repeated pitch class in a pitch-class set is not included in the calculation of an interval vector. In the first example the hand with only four distinct entries ranks higher than the hand with five distinct entries, not because it contains a pair as in traditional poker, but because it includes an ic6.

Figure 1.

Hand	Interval Vector	Rank
(C#, E, F, F, B) or (A, 4, 5, 5, J)	<111111>	higher
(C, E, F, G, G#) or (K, 4, 5, 7, 8)	<212320>	lower

If two hands are equal with respect to ic6, the higher rank will be assigned to the hand with the greater number of ic5's. Again, if the hands are equal with respect to ic5, the hand with the greatest number of ic4's will rank higher, and so forth. If two hands have identical interval vectors, the higher rank is assigned to the hand that contains the highest card. If the highest cards are identical, the next highest cards are compared, and so forth down to the final card. Figure 2 shows a specific case.

(C#,D,F,G,Ab) or (A,2,5,7,8) representing the interval vector <212122> ranks higher than (C,C,D,F#,G#) or (K,K,2,6,8) representing the interval vector <020202>, since the first hand contains two ic5's, while the second contains no ic5's, although both hands contain two ic6's. However, the hand (C,F,F#,A,B) or (K,5,6,9,J) representing the same interval vector as the winning hand above <212122> ranks highest of these three hands, since the B or Jack ranks higher than A^{b} or 8. If the hands are identical in all respects, there is a tie. Perhaps, if using a deck of cards showing clefs rather than suits, the hand containing the card in the highest register could break the tie. House rules can be developed for these rare situations if the players do not want to chance a tie game.

Figure 2.

<u>Hand</u>	<u>Interval Vector</u>	<u>Rank</u>
(C#, D, F, G, Ab) or (A, 2, 5, 7, 8)	<212122>	higher
(C, C, D, F#, G#) or (K, K, 2, 6, 8)	<020202>	lower
(C, F, F#, A, B) or (K, 5, 6, 9, J)	<212122>	highest

To determine the potential strength of a particular hand before placing a wager, a player must be acquainted with the distribution of ic6 within all possible interval vectors for pentachords, tetrachords, trichords, and dyads, since a five-card Pitch-Class Poker hand could contain a pair (thus representing a tetrachord) or three, or four of a kind (thus representing a trichord or dyad). Examining a list of interval vectors reveals that of the 81 distinct interval vectors of collections of two, three, four, or five pitch classes, nine contain two ic6's, and 38 contain a single ic6.

A traditional list of interval vectors does not contain enough information for a Pitch-Class Poker player, however, since each interval vector can be represented by a large number of poker hands depending on the cardinality of the pitch-class set represented by the hand as well as the symmetric properties exhibited by that set. The list given in Figure 3 ranks the 81 possible interval vectors according to the rules for Tritone Trump. Each entry shows an interval vector, the potential number of hands that represent that interval vector, and the percentage of hands that are lower than a hand representing the given interval vector. For reference the set name and prime form for each interval vector are also listed, or in the case of the Z-related sets both set names and prime forms associated with the interval vector are listed.

Figure 3. Ranking of Pitch-Class Poker Hands for Tritone Trump.

Vector	Set Name	Prime Form	Hand Rank	# of Hands	% Hands Below
310132	5-7	(0, 1, 2, 6, 7)	1	24576	98.56%
220222	5-15	(0, 1, 2, 6, 8)	2	12288	97.85%
212122	5-19	(0, 1, 3, 6, 7)	3	24576	96.41%
200022	4-9	(0, 1, 6, 7)	4	9216	95.87%
122212	5-28	(0, 2, 3, 6, 8)	5	24576	94.44%
114112	5-31	(0, 1, 3, 6, 9)	6	24576	93%
040402	5-33	(0, 2, 4, 6, 8)	7	11288	92.29%
020202	4-25	(0, 2, 6, 8)	8	9216	91.75%
004002	4-28	(0, 3, 6, 9)	9	4608	91.48%
211231	5-20	(0, 1, 3, 7,8)	10	24576	90.04%
122131	5-29	(0, 1, 3, 6, 8)	11	24576	88.61%
221131	5-14	(0, 1, 2, 5, 7)	12	24576	87.17%
202321	5-22	(0, 1, 4, 7, 8)	13	12288	86.45%
121321	5-30	(0, 1, 4, 6, 8)	14	24576	85.02%
113221	5-32	(0,1, 4, 6, 9)	15	24576	83.58%

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032221	5-34	(0, 2, 4, 6, 9)	16	12288	82.87%
212221	5-Z18/5-Z38	(0, 1, 4, 5, 7) or (0, 1, 2, 5, 8)	17	49152	80%
131221	5-24	(0, 1, 3, 5, 7)	18	24576	78.56%
311221	5-6	(0, 1, 2, 5, 6)	19	24576	77,13%
123121	5-25	(0 2, 3, 5, 8)	20	24576	75.69%
222121	5-Z12/5-Z36	(0, 1, 3, 5, 6) or (0, 1, 2, 4, 7)	21	36864	73.54%
321121	5-5	(0, 1, 2, 3, 7)	22	24576	72.1%
110121	4-16	(0, 1, 5, 7)	23	36864	69.95%
200121	4-8	(0, 1, 5, 6)	24	18432	68.87%
210021	4-6	(0, 1, 2, 7)	25	18432	67.8%
122311	5-26	(0, 2, 4, 5, 8)	26	24576	66.36%
221311	5-13	(0, 1, 2, 5, 8)	27	24576	64.93%
213211	5-16	(0, 1, 3, 4, 7)	28	24576	63.49%
231211	5-9	(0, 1, 2, 4, 6)	29	24576	62.06%
223111	5-10	(0, 1, 3, 4,6)	30	24576	60.62%
322111	5-4	(0, 1, 2, 3, 6)	31	24576	59.18%
012111	4-27	(0, 2, 5, 8)	32	36864	57.03%

Figure 3. (Con't.)

Vector	Set Name	Prime Form	Hand Rank	# of Hands	% Hands Below
102111	4-18	(0, 1, 4, 7)	33	36864	54.88%
111111	4-Z15/4-Z29	(0, , 4, 6) or (0, 1, 3, 7)	34	73728	50.57%
210111	4-5	(0, 1, 2, 6)	35	36864	48.42%
112011	4-13	(0, 1, 3, 6)	36	36864	46.27%
100011	3-5	(0, 1, 6,)	37	14976	45.39%
020301	4-24	(0, 2, 4, 8)	38	18432	44.32%
232201	5-8	(0, 2, 3, 4, 6)	39	12288	43.6%
030201	4-21	(0, 2, 4, 6)	40	18432	42.52%
112101	4-12	(0, 2, 3, 6)	41	36864	40.37%
010101	3-8	(0, 2, 6)	42	14976	39.49%
002001	3-10	(0, 3, 6)	43	7488	39.06%
000001	2-6	(0, 6)	44	336	39.04%
032140	5-35	(0, 2, 4, 7, 9)	45	12288	38.32%
122230	5-27	(0, 1, 3, 5, 8)	46	24576	36.89%
132130	5-23	(0, 2, 3, 5, 7)	47	24576	35.45%

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021030	4-23	(0, 2, 5, 7)	48	18432	34.37%
202420	5-21	(0, 1, 4, 5, 8)	49	24576	32.94%
212320	5-Z17/5-Z37	(0, 1, 3, 4, 8) or (0, 3, 4, 5, 8)	50	24576	31.5%
222220	5-11	(0, 2, 3, 4, 7)	51	24576	30.07%
101220	4-20	(0, 1, 5, 8)	52	18432	28.99%
012120	4-26	(0, 3, 5, 8)	53	18432	27.91%
021120	4-22	(0, 2, 4, 7)	54	36864	25.76%
111120	4-14	(0, 2, 3, 7)	55	36864	23.61%
010020	3-9	(0, 2, 7)	56	7488	23.17%
101310	4-19	(0, 1, 4, 8)	57	36864	21.02%
322210	5-3	(0, 1, 2, 4, 5)	58	24576	19.58%
102210	4-17	(0, 3, 4, 7)	59	18432	18.51%
201210	4-7	(0, 1 4, 5)	60	18432	17.43%
332110	5-2	(0, 1, 2, 3, 5)	61	24576	16%
121110	4-11	(0, 1, 3, 5)	62	36864	13.84%
211110	4-4	(0, 1, 2, 5)	63	36864	11.69%
001110	3-11	(0, 3, 7)	64	14976	10.81%
100110	3-4	(0, 1, 5)	65	14976	9.94%

Figure 3. (Con't.)

Vector	Set Name	Prime Form	Hand Rank	# of Hands	% Hands Below
122010	4-10	(0, 2, 3, 5)	66	18432	8.86%
011010	3-7	(0, 2, 5)	67	14976	7.99%
000010	2-5	(0, 5)	68	672	7.95%
000300	3-12	(0, 4, 8)	69	2496	7.8%
432100	5-2	(0, 1, 2, 3, 4)	70	12288	7.09%
212100	4-3	(0, 1, 3, 4)	71	18432	6.01%
221100	4-2	(0, 1, 2, 4)	72	36864	3.86%
101100	3-3	(0, 1, 4)	73	14976	2.98%
020100	3-6	(0, 2, 4)	74	7488	2.55%
000100	2-4	(0, 4)	75	672	2.51%
341000	4-1	(0, 1, 2, 3)	76	18432	1.43%
111000	3-2	(0, 1, 3)	77	14976	0.56%
001000	2-3	(0, 3)	78	672	0.52%
210000	3-1	(0, 1, 2)	79	7488	0.08%
010000	2-2	(0,2)	80	672	0.04%
100000	-21	(0, 1)	81	672	0%

To become an accomplished player requires subtle strategic thinking, with an understanding of the abstract subset and superset relations among pitch-class sets. Various inclusion relations have been defined in the theoretical literature, focusing mainly on the subset structure of a set-class or the relations between two set-classes.[3] The superset structure of a specific set-class has been addressed, but in less detail. The practical value of understanding the superset structure as well as the subset structure of a collection is clear, for both structures are engaging musically and analytically. When playing Pitch-Class Poker, the superset structure for a given collection of pitch classes is as critical or even more critical than the subset structure of a collection. To introduce these concepts, a practical, intuitive approach for developing a winning strategy in Tritone Trump will be discussed.

In draw poker, after the initial wagers are placed, each active player must determine which cards might be discarded to improve the original hand with replacements from the deck. The player must determine which subset of the original hand should be retained to increase the odds of obtaining more tritones and large intervals. Intuitively, a player knows that a duplicate pitch class forming a pair should be discarded, retaining the card that represents the pitch class in a higher register, if house rules favor higher pitches. Furthermore, the player realizes that any pair of cards that represent a tritone should be retained, and possibly any pair that represent ic5. If each of the four intervals a card forms with the other cards in the hand is less than ic5, the card might be replaced in hopes of obtaining a ic6 or ic5. If each of the four intervals is less than ic4, the card will very likely be discarded.

These intuitive guidelines for determining whether a card should be retained in the hand or discarded are based on the probabilities for obtaining each interval-class with a given card when choosing at random from the other 47 cards in the deck. There are three other cards that duplicate its pitch class, four cards that form ic6 with the given card, eight cards that form ic5 with the given card, as well as eight cards each that form ic4, ic3, ic2, and ic1 with the given card. Of the 47 intervals that could be formed with a given card, 27 are smaller than ic4.

The advice to discard any card that forms only intervals smaller than ic4 with the other cards in the hand is based on the knowledge that odds are slightly less than even (43% or 20 out of 47) for obtaining an interval greater than ic3 when choosing a second card at random. Thus, if all the intervals formed by a particular card in the hand are less than ic4, it will probably be worth the risk to replace the card with a new card from the deck. Moreover, the advice to retain a card that forms ic5 or greater is sound since only 12 of

47, or 25%, of the cards, will produce ic5 or ic6. These guidelines may be sufficient to play the game, but a curious or cautious player may wish to explore the issue further before placing any substantial wagers.

One might consider the numerical odds for improving the hand based on the cards the player chooses to retain to confirm that appropriate cards were discarded from the hand. That is, the player might examine all the potential five-note supersets for the subset represented by the cards retained after the initial discard. This is not the same as considering all the subsets of the original five-note hand.

Intuitively, one expects the subset and superset relations to be similar. However, as can be shown by example, there are differences, and the differences are significant. Given two sets X and Y such that X is smaller than Y, the number of times X is contained in Y is defined as the number of different subsets of type X that are contained in a single instance of set Y; this is Lewin's embedding function. The number of times Y is a superset of X is defined as the number of times sets of type Y occur when additional elements are added to a single instance of set X; this is Lewin's covering function.[4] A specific six-note collection such as the whole-tone scale includes two augmented triads as subsets, yet a specific augmented triad has only one six-note superset that is a whole-tone scale.[5]

Appreciating the distinction between the process of finding supersets and the process of finding subsets is critical in understanding atonal theory and in playing pitch-Class Poker. The intuition that the embedding and covering functions are opposite or complementary processes may be misleading, yet is not entirely false since the covering function of two sets is equal to the embedding function of the complements of the same two sets.[6] The covering and embedding functions also correspond when both equal zero, but when their values are greater than zero they will not necessarily be equal.

An example from Pitch-Class Poker will help clarify the situation. If a player is dealt the hand (C,D,E,B^b,B^b) with the interval vector <030201> ranking higher than only 42.5% of the other Tritone Trump hands and follows the intuitive strategy outlined above, the player will choose to keep the pitches that form ic6 in the hand (the E and the Bb in the higher register) and choose to discard the Bb in the lower register. The player must decide what to do with the C and the D. The C forms ic4 with the E and ic2 with the D and the B^b, the D forms ic4 with the B^b and ic2 with the C and the E. Assume the player retains both the C and D in the hand and thus draws one additional card from the remaining 43 unknown cards in the deck. The deck will actually contain fewer than 43 cards since each player holds five cards, but the probability for improving the hand (C,D,E,Bb) when drawing a single new card is based on the number of unknown cards in the deck, not the actual number of cards left to deal. Figure 4 shows the potential hands that the player may obtain after receiving a new card.

The player may be dealt one of the three remaining C's, D's, or E's in the deck, or one of the two remaining Bb's. In this case there would be no change in the interval vector. The player may receive one of four remaining C#'s, changing the interval vector to <232201>, and improving the hand only 1%, to rank higher than just 43.6% of all hands. The player may be dealt one of the four remaining B's or four remaining D#'s, changing the interval vector to <231211>, and improving the hand to rank higher than 62.1% of all hands. The player may be dealt one of four A's or four F's, changing the interval vector to <131221>, and improving the hand's rank to 78.6%. The player may be dealt one of four G's, changing the interval vector to <032221>, and improving the hand's rank to 82.9%. Finally, the player may be dealt one of four F#'s or one of four G#'s, changing the interval vector to <040402>, and improving the hand's rank to 92.3%. In 15 of 43 cases the hand will not improve substantially, in 8 of 43 cases the hand will improve somewhat, and in 20 of the 43 cases the hand will improve substantially.

Figure 4.

Hand	Interval Vector	Rank	Odds
(C, D, E, Bb)	<030201>	42.5%	11/43
(C,C#,D,E,Bb)	<232201>	43.6%	4/43
(C, D, E, Bb, B) or (C, D, D#, E, Bb)	<231211>	62.1%	8/43
(C, D, E, A, Bb) or (C, D, E, F, Bb)	<131221>	78.6%	8/43
(C, D, E, G, Bb)	<032221>	82.9%	4/43
(C, D, E, F#, Bb) or (C, D, E, G#, Bb)	<040402>	92.3%	8/43

The procedure outlined above, adding each of the remaining pitch classes to the existing tetrachord, call it X=(C,D,E,B^b), is a practical example of finding all the pentachords, call them Y, for which Lewin's covering function COV(X,Y) is greater than 0.[7] Some of the pentachords may be formed by more than one additional pitch-class, for example, the pentachord representing the interval vector <231211>. Specifically, X=(B^b,C,D,E) may be covered by Y1=(B^b,B,C,D,E) or Y2=(B^b,C,D,D#,E), two distinct forms of set-class Y. In this case COV(X,Y)=2.

The situation becomes more complicated if the player considers replacing tow cards in the existing hand. The C or the D might be discarded in addition to the duplicate B^b. In both cases, the remaining trichord would be set-type 3-8, (D, E,B^b) or (C,E,B^b). Assume C is discarded; the player retains the trichord (D,E,B^b), with the interval vector <010101>, and draws two new cards. This trichord ranks higher that 39.5% of the hands in Tritone Trump, only 3% lower than the original rank of 42.5%.

The player has not sacrificed much to begin with a trichord rather than a tetrachord before the draw. It remains to be seen if the odds for improving the hand will increase after starting with the trichord. There are 78 distinct pairs of pitch classes in the 43-card deck that remains. Finding all the sets that cover 3-8 by including each distinct pair with the original set, as in the example above, is tedious. Using a computer generated list that indicates the number of ways a smaller set can be covered by a larger set saves time.

Figure 5 shows vectors that represent the covering function for trichords and tetrachords as well as trichords and pentachords, while Figure 6 shows vectors representing the covering function for tetrachords and pentachords.[8] These tables are read by first locating the smaller set, the set to be covered, or the set represented by the cards retained in the player's hand, in the left-hand column of the table. For example, set-type 3-12 is at the bottom of Figure 5. Next, one must locate non-zero entries in the vector representing the covering function (for example, the 6 and the 3 in the Tetrachord Covering Function for set-type 3-12). The 6 is found in the 19th place of the vector, while the 3 is found in the 24th place of the vector. This indicates that set 4-19 covers 3-12 six ways, and set 4-24 covers set 3-12 three ways. The covering functions for other trichords and tetrachords can be read from the vectors on these tables in a similar manner. Once players are aware of the potential covering sets for the set represented by the cards they choose to retain in the playing hand, those sets can be located in Figure 3, and the player can determine if there is a good chance of improving the hand by retaining those particular cards. I caution you that each covering set can be formed by a variety of cards, since the full deck contains four versions of each pitch-class.

Figure 5. Pentachord and Tetrachord Covering Function for Trichords.

Set Name	Prime Form	Tetrachord Covering Function	Pentachord Covering Function
3-1	012	22022100000000000000000000000	34244221202022100000000000000000000212
3-2	013	11100000011111000000000000001	23222001131110021111002121111010000100
3-3	014	01110010000100101110000000000	10310201122011031210310001010112000111
3-4	015	00011011001001010011000000000	00211320101111101203311101201100000012
3-5	016	00001101100010110100000000001	00012250110112110142010100111110000201
3-6	024	02000000002000000000220100000	10200002402120000000002402200200321200
3-7	025	00010000011010100000011001100	00110100121102000010003131212112012111
3-8	026	00001000000100110000100110101	00011111210021210111000212041211310001
3-9	027	00000100000002020000022000000	00002020002004101202004200204200013200
3-10	036	00000000000220000200000000210	00020001020100020220010022022082010202
3-11	037	00000000000001001111010001101	00001000002010021112311121311113011112
3-12	048	00000000000000000060000300000	00000000000060003000630006000600300030

Figure 6. Pentachord Covering Function for Tetrachords.

Set Name	Prime Form	Pentachord Covering Functioning
4-1	0123	22022000000000000000000000000000000000
4-2	0124	11100001101010000000000000000000000100
4-3	0134	10200000002000000210000000000000000000
4-4	0125	01110100001001000000000000000000000011
4-5	0126	00011110100001010000000000000000000001
4-6	0127	00002020000000200000000000000000000200
4-7	0145	00200200000000000200200000000000000000
4-8	0156	00000220000100000002010000000000000000
4-9	0167	00000040000000000004000000000000000000
4-10	0235	02000000020000000000020200000000000000
4-11	0135	01100000010010000000000110110000000000
4-12	0236	00010001010000010100000000101001000000
4-13	0136	00010000010100000001000000100010100000100
4-14	0237	00001000000100000110100100010100000000
4-Z15	0146	00000100110001000010000000001010100000

4-16	0157	00000010000000110010100010000110000000
4-17	0347	00000000002000020000200000000002000000
4-18	0147	00000000000000010110010000000011000101
4-19	0148	00000000000010001000210001000100000010
4-20	0158	00000000000000000022000000200000000002
4-21	0246	00000001200000000000000200000000210000
4-22	0247	00000000001000000000001100100100011100
4-23	0257	00000000000020000000002000002000002000
4-24	0248	00000000000020000000000002000200200000
4-25	0268	00000000000002000000000000040000200000
4-26	0358	00000000000000000000000202000002001010
4-27	0258	00000000000000000000000011011011010001
4-28	0369	00000000000000000000000000000008000000
4-Z29	0137	00001000000001001001100011001000000000

In the example from Pitch-Class Poker, the player is specifically interested in the pentachords and the tetrachords that cover (D,E,B^b). There are 36 pentachords and 9 tetrachords that cover any particular trichord. Some of these may be duplications, as in the specific example under consideration now. Figure 5 reveals that (D,E,B^b), set-type 3-8, can be covered by the supersets 4-5, 4-12, 4z15, 4-16, 4-21, 4-24, 4-25, 4-27, 4z29, 5-4, 5-5, 5-6, 5-7, 5-8, 5-9 twice, 5-10, 5-13 twice, 5-28 four ways, 5-29, 5-30 twice, 5-31, 5-32, 5-33 3 ways, 5-34, and 5z38. These sets can now be located in Figure 3 to determine which covering sets will improve the hand dramatically and which will improve the hand only slightly. All the covering sets will rank higher than the trichord 3-8 with the interval vector <010101>.

The probability for obtaining each of these covering sets is calculated by first considering the odds for obtaining any particular pair when drawing two additional cards at random from the remaining 43. There are 903=(43x42)/2 unordered pairs of cards in the 43-card deck that remains. To calculate the odds for obtaining a particular set, for example 5-7 (the highest ranking hand in Tritone Trump), one must first determine which pair or pairs of pitch classes will complete a version of set-type 5-7 when included with the original D, E, and B^b. Since 5-7 covers 3-8 just once, the only possible pair is E^b and A. All four E^b's and all four A's remain in the deck of 43 unknown cards, thus there are 16 ways to find this pair. Thus, about 1.8% of the 903 different hands that could result when the player draws two new cards will represent the interval vector <310132> or set-type 5-7. When the player discards only one card, it is not possible to obtain an equivalent hand.

Similar calculations can be performed for all 45 sets that cover 3-8 to reveal the following statistics: 26% of the new hands will rank higher than 90%, 14% will rank between 80% and 90%, 8% rank between 70% and 80%, 17% rank between 60% and 70%, 14% rank between 50% and 60%, while 17% rank between 40% and 50%. About 3% of the new cards dealt will duplicate pitch classes from the original trichord, and the player will keep the interval vector <010101>. These are better odds than when four cards were retained and just the duplicate B^b exchanged.

Finally, the odds for improving the hand are calculated when the player retains two cards, E and B^b, and discards three cards, B^b, C, and D. While there are 12,341=(43x42x41)/2(3x2) potential hands when retaining two and drawing three new cards, there are 175 sets of cardinality three, four, or five that cover the dyad 2-6, including duplications of the same set-types. There are 41 different set-types or hands that rank higher than 2-6 (39.04%) in Tritone Trump, each of which covers set-type 2-6 at least once.

Calculating the odds for obtaining each of these 41 sets will be tedious, so only the odds for the 10 sets that rank above 90% will be computed. 5-7 again serves as an example. There are eight ways for some form of the highest ranking set-type to cover the dyad E-B-flat: (4,5,6,10 11) will occur 64 times, (3,4,5,9,10) will occur 64 times, (4,5,9,10 11) will occur 64 times, (2,3,4,9,10) will occur 48 times, (3,4,5,10 11) will occur 64 times, (3,4,8,9,10) will occur 64 times, (0,4,5,10,11) will occur 48 times, and (3,4,9,10,11) will occur 64 times, for a total of 480 hands that represent set-type 5-7 and include the original B^b and E, almost 4% of the 12,341 potential hands.

Thus the odds of obtaining a hand representing set-type 5-7 are greater when discarding three cards than when discarding only two. Repeating these calculations for the other nine high-ranking sets shows that 2,816 of 12,341 hands will rank higher than 90%, but this is only 23%, less than the percentage of hands ranking above 90% when just two cards were discarded. Thus, although the chances of obtaining the best possible hand are greater when discarding three cards, the chances of obtaining a hand ranking above 90% are greatest when just two cards are discarded.

Calculating the exact odds for improving a given hand is tedious, and the game can be played successfully with only an intuitive understanding of which cards to retain and which to discard. But finding the exact probabilities for a few examples helps develop a stronger sense of how pitch classes and interval classes interact, and aids the player in understanding theoretical inclusion relations.

An alternate version of the game, called Semitone Sweep, in which the ideal sonority contains as many semitones or small intervals as possible can also be played. For this game the basic techniques and strategy resemble Tritone Trump, however the player seeks to maximize the number of ic1's in the hand, or the number of small interval classes, rather than maximizing ic6, or large interval classes.

To conclude, I would like to remind you of the musical implications of the game. Playing Tritone Trump is similar to comparing sonorities that include five pitches chosen at random from a gamut of four octaves. In this game the ideal sonority contains as many tritones as possible, or as many large interval classes as possible. In the draw poker version of the game each player may replace one or more of these random pitches by other almost random pitches hoping to increase the number of desirable interval classes in the sonority.

To become adept at either game the player must learn to recognize interval classes and learn to modify a given collection to maximize a specific interval class. The random method of choosing replacement pitches will not insure players an ideal sonority, but will develop an understanding of interval vectors, supersets, and subsets regardless of whether the game is finished with the highest ranking hand.

NOTES

[1]Albert H. Morehead and Geoffrey Mott-Smith, eds., *Hoyle's Rules of Games*, second revised edition (New York: New American Library, 1946, 1983), 34-63.

[2]Instructions for calculating the interval vector of a pitch-class set may be found in Allen Forte, Th*e Structure of Atonal Music*, (New Haven: Yale University Press, 1973), 13-15.

[3]Examples may be found in David Lewin, *Generalized Musical Intervals and Transformations* (New Haven, CT: Yale University Press, 1987); Allen Forte, *The Structure of Atonal Music* (New Haven, CT: Yale University Press, 1973); and Robert Morris, *Composition with Pitch-Classes* (New Haven, CT: Yale University Press, 1987).

[4]David Lewin, *Generalized Musical Intervals and Transformations* (New Haven, CT: The Yale University Press, 1987), 105-120.

[5]Formula 1): Given n≥j and 0!=1, the number of subsets of cardinality j in a (super)set of cardinality n (n things taken j at a time) is: C(n,j)=n!/(j!(n-j)!).

Formula 2): Given n≥j and 0!=1, the number of supersets of cardinality n for a (sub)set of cardinality j (12-j things taken n-j at a time) is: P(n,j)=(12-j)!/((n-j)!(12-n)!).

[6]COV (X,Y)=EMB (comp (X). Lewin, 1987, p.120; Morris, 1987, p. 90.

[7]David Lewin, *Generalized Musical Intervals and Transformations* (New Haven, CT: Yale University Press, 1987), 120.

[8]To save space the vectors for dyads covered by trichords, tetrachords, or pentachords are not included here. For a complete table of covering functions see also Robert Morris, *Composition with Pitch-Classes* (New Haven, CT: Yale University Press, 1987), 330, note 46; or Bo Alphonce, *The Invariance Matrix*. Ph.D. dissertation, Yale University, 1974, contains a subset and superset list. A computer generated list by Stephen Haflich in 1979 differs from the Alphonce list in that it distinguishes among the number of sets embedded in a nexus set and the number of ways a nexus set can be covered by a superset. For the present context this is a crucial distinction.