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In recent years many music programs have recognized the importance of strengthening course offerings in 20th-century music. Rather than forcing the student to wait until the third or fourth years of a music program to take a course in 20th-century music, many schools have increased coverage of this repertory in the basic undergraduate theory program. This set of core theory courses in many institutions is generally charged with introducing the student to a wide range of music literature from as many different style periods as possible.

At this relatively early stage of college-level instruction, and especially after a thorough and systematic treatment of tonal music (often occupying as many as three or four semesters), the typical one-semester or shorter introduction to 20th-century music frequently strikes the student (and instructor) as superficial and limited in scope. Of course, some of this reaction is engendered simply by the stylistic variety and eclecticism found in the 20th- century repertory—characteristics that, for the instructor, may make course design especially problematic and, for the student, may create difficulties in the organization of such a large body of theoretical information.

Nevertheless, a considerable proportion of student difficulties result, I believe, from the lack of precise theoretical and analytical tools traditionally available to students at this level. We can only sympathize with those students who, after learning to describe tonal harmonies in a variety of ways, no longer possess a method that allows them to discuss in precise terms the properties of pitch collections found in examples of 20th-century music. The curious students discover, all too soon, that terms like *polychord* or *mixed-interval chord* can only take them so far analytically. They soon feel the need for a systematic method of presenting analytical conclusions. Set theory, properly distilled and efficiently presented, can provide such a method.

My purpose in this paper will be to suggest some techniques that I have found particularly useful in confronting the problems instructors typically face in presenting elementary set theory to college analysis classes.[1] We will explore how the authors of various undergraduate theory textbooks deal with some of the pedagogical issues involved, and finally I will present sample analyses that could be undertaken with undergraduate students who possess a basic knowledge of set-theoretic principles.

I believe that several fundamental problems confront the instructor who attempts to present set theory to a beginning class in 20th-century theory or analysis. I am sure many instructors have become frustrated when, in the presentation of an analytical method, the class seemed to be "out of touch" with the actual sounds of the piece. We tell our students that analysis must somehow illuminate the music, which must therefore be the necessary starting point. Nevertheless, we discover that students sometimes lack a basic familiarity with the music to which set-theoretic methods are most frequently applied. Given the traditional contents of many of today's concert programs as well as the nature of pre-college instrumental and vocal instruction, our students often have not listened to or performed this music to any great degree.

Of course, it is not the theory faculty alone who have the responsibility for actively promoting 20th-century music; we can, however, help in a number of ways. For instance, when discussing the elements of music in a beginning theory or rudiments class, we might use examples from 20th-century compositions. Or we might take advantage of opportunities in the classroom to show students how compositional techniques of earlier style periods carry over into the 20th century. At the very least, we must always be ready to encourage students to study and perform 20th-century music outside of their classroom responsibilities.

Another problem involves the very nature of set-theoretic concepts and terminology. These may seem to many students novel, esoteric, and unrelated to anything they have encountered in their previous musical training.[2] They may have difficulty thinking about musical relationships in terms of integers, the traditional symbols of set theory. A particular stumbling block for some students is modulo 12 arithmetic. (A discussion of "clock" arithmetic often helps here.) A sensitive instructor should be aware of not only these specific difficulties, but of the frustration emanating from them as well. An extra bit of patience can do much to carry the reasonably prepared music student over these obstacles.

The last problem I would like to mention here involves course organization. The great amount of time spent in developing set-theoretic concepts in the classroom and refining students' computational skills often means less time spent with actual music—a sacrifice many instructors are unwilling or unable to make. Given many students' unfamiliarity with 20th-century music, instructors often find the initial 20th-century theory course to be more a general introduction to this literature rather than a more detailed analytical study. In a situation such as this, we must ask whether it is wise to trade the more intuitive aspects of musical awareness for theoretical rigor. The following remarks will demonstrate my contention that, in a class situation, it is possible to make some of the seemingly abstract concepts of set theory more musically relevant and thereby avoid an unnecessary trade-off between theory and musical reality.

One of the simplest ways to introduce set-theoretic concepts is to link them in the student's mind to concepts covered earlier in the theory program. Several of the available undergraduate texts dealing with set theory adopt this strategy. Ralph Turek's *The Elements of Music* presents transformations of interval cells in terms of traditional motivic operations, although he claims that greater freedom exists in the manipulation of interval cells.[3] This approach reminds us of Schoenberg's emphasis on the motive as a way of understanding compositional process not only in tonal music, but in atonal and serial music as well.[4] Earl Henry's *Music Theory* also begins with a discussion of basic operations, defined as transposition, inversion, inclusion, and complementation.[5]

One may go further, however, in drawing connections between the principles of set theory and those of tonal theory. The postulate of the unordered set, for example, can be related to the function of a scale in a tonal piece. Students can easily see that whether or not its pitches appear in direct order, a scale serves as a "reservoir" of pitches for a given passage. For many examples of 20th-century music, this idea can be expanded to any pitch collection with provision made for the application of transposition and inversion to subsets and supersets of the original collection.[6] Another link between set theory and traditional harmonic theory can be found in the relationship of a pitch-class (pc) set in normal order to a root-position triad: both structures exhibit a similar arrangement with the smallest interval from first to last pcs.[7] The concept of interval classes can easily follow an elementary presentation of intervals and their inversions. The typical introduction to triad types actually involves chord analysis through the use of interval vectors, since we generally require students to learn the total interval content of these sonorities. Even the inclusion relation has its analog in tonal theory with the relationship of the leading-tone triad to a dominant-seventh of the same key. Of course, examples of transposition and inversion abound in the tonal literature.

A simple step for classes beginning a study of set theory is the initial avoidance of integer notation. Although making the issues of octave and enharmonic equivalence somewhat more difficult to manage, this strategy has the advantage of keeping students closer to the actual musical relationships. For the same reason, so-called integer maps (complete integer representations of pieces) should probably be avoided in the earlier stages of instruction. Both Turek and Bruce Benward, the latter in his Music in Theory *and Practice,* determine normal orders with pc sets in staff notation.[8]

Interval vectors, of course, can easily be derived from traditionally notated sets, as can set types. The latter designation uses integers, but, as in Turek and Benward, these integers represent intervals above the initial pc of the normal order rather than pcs.[9]

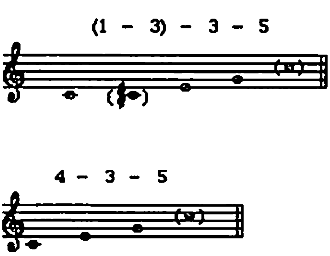
One of the most effective tools in the teaching of set theory is the successive-interval array or SIA, first discussed comprehensively in the literature by Richard Chrisman.[10] The SIA possesses two basic qualities of any efficient analytical tool: it is simple, and it is powerful. Figure 1 shows two pc sets in normal order with their corresponding SIAs, which are merely representations of the successive intervals of the sets, including the interval from last to first pc.[11]

Figure 1.

Although more cumbersome, the SIA can be used in place of the set type designation or the Forte name to represent a class of sets equivalent under transposition or inversion. The SIA can also greatly facilitate the determination of normal order, since this form of the set always begins with the pc to the right of the largest interval of the SIA. When this largest interval is then inverted, it becomes the smallest boundary interval of the set, thereby guaranteeing normal order.

The inclusion relation can easily be understood through the use of the SIA. Figure 2 shows the omission of the C# from the first set of figure 1.

Figure 2.



This results in the combination of the two intervals formed by this pc; that is, C-sharp no longer subdivides the interval between C and E. The resulting new SIA denotes one of the subsets of the original collection. The advantage of this treatment of inclusion is two-fold: subsets of a collection can be systematically determined by manipulation of the SIA alone, and the pcs forming a particular subset are readily at hand. The latter situation greatly assists in the comprehension of nonliteral inclusion, since the student can easily go beyond the name or SIA of a subset to its actual pcs in the larger set. This leads the student from a relatively abstract analytical construct to a focus on the actual notes in a passage.

The inversional relation is one of the more difficult aspects of set theory to teach. Fortunately, the inversional equivalence of two pc sets is fairly easy to describe using SIAs. As previously noted the two sets of figure 1 are inversionally related. Note the reversal of the first three entries of the SIA. Students familiar with the traditional process of motivic inversion will understand the notion of directional reversal in an interval succession. Given the ascending direction of a normal order, they should then be able to see how placing a set in normal order, after deriving it through the reversal of interval directions, will result in the kind of SIA correspondence shown in figure 1. Of course, the important point to note is that once all of this is explained to students, they will be able to uncover inversionally equivalent sets at a glance.

Recognizing the inversional relation, however, sidesteps the issue of the true nature of inversion. As you may recall, Forte sets up a fixed, one-to-one correspondence (mapping) between each pc integer and an inverse, with pcs 0 and 6 mapping onto themselves.[12] Thus, 0 inverts to 0, 1 to 11, 2 to 10, and so on. In this way, to derive a particular inverted form of a pc set, one might need to transpose after inverting the pcs of the original set. An alternative to this approach involves the reliance on the inversional index to describe the relation between inversionally equivalent sets.[13] As figure 3 shows, the two sets of figure 1 (rewritten in integer notation with C=0) can be rearranged so that the sum of all corresponding elements is the same. This sum is the inversional index.

Figure 3.

0	1	4	7
<u>7</u>	<u>6</u>	<u>3</u>	<u>0</u>
7	7	7	7

By establishing a particular index, one can derive an inverted form of a pc set by simply subtracting each of its elements from the index without recourse to transposition.

In addition, knowing the index provides the inversional centers or axes, that is, the single pcs or semitone dyads a tritone apart that bisect all intervals formed by corresponding elements of the inversionally equivalent sets. Figure 4 shows the two inversional centers of the sets in figure 1 and how they relate to the other dyads formed between the sets.[14]

Figure 4.

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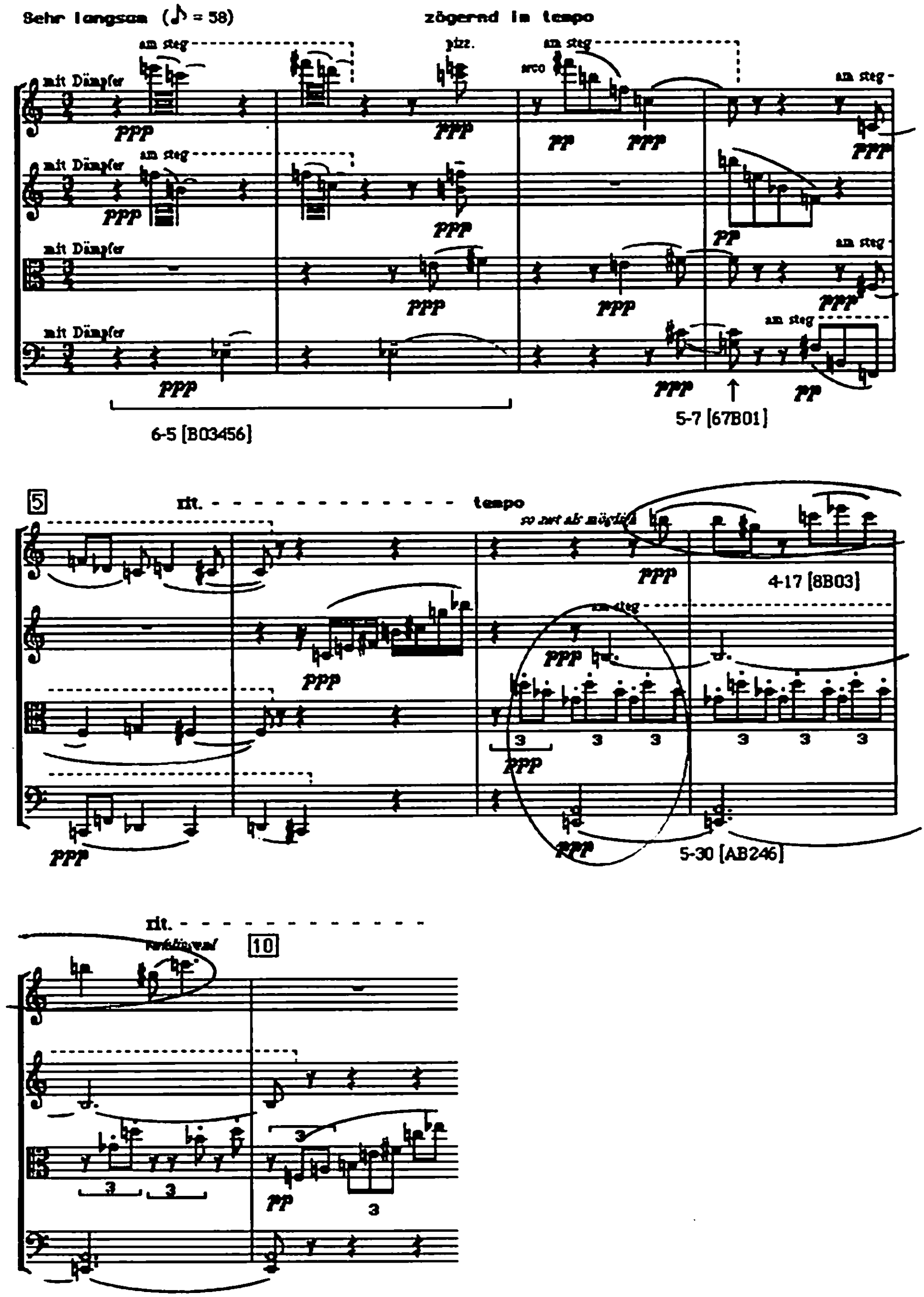
┌─3─┐ ┌─3─┐        ┌─2─┐ ┌─2─┐
0   (34)   7        7   (9A)   0


┌─2─┐ ┌─2─┐        ┌─3─┐ ┌─3─┐
1   (34)   6        6   (9A)   1
```

Of course, in sets having fewer than 12 elements, one or both of the inversional axes may not actually be present. Nevertheless, as David Lewin has pointed out, inversional axes are often singled out for special contextual emphasis.[15] A familiar example illustrating the significance of inversional axes is the second movement of Webern's *Piano Variations*, op. 27, in which pairs of row forms invert about the axis pcs A and E-flat. In this example, thinking about the row-form pairs in terms of inversion about the initial pc of the row followed by transposition obscures the more fundamental relationship and adds an additional, abstract, and unnecessary step to the analytical model.

At this point I would like to present three analyses that apply the techniques discussed above and that would be suitable for a beginning class in set theory. The first excerpt, shown in figure 5, consists of the opening 10 measures from the fourth movement of Webern's *Five Movements for String Quartet*, op. 5. In the figure I have labeled certain pitch collections with Forte names.[16]

Figure 5. Webern, Op. 5, No. IV, mm. 1-10.



This passage provides excellent examples of inclusion relations, as well as the more general feature of sectional contrast based on interval content.

At first glance, the middle section of the piece, measures 7 through 9, embodies contrasts of primarily rhythm and articulation. Set structure, however, acts as a further means of differentiation. The melodic set, 4-17, is a subset of neither pc sets 5-7 nor 6-5 found in measures 1 through 6. The accompanimental set, 5-30, is also not a subset of 6-5; moreover, it shares no common interval vector entries with 5-7, a situation described by Forte as maximum intervallic dissimilarity (the R_0 relation).[17]

The second violin line of measure 6 and the viola line of measure 10, both forms of pc set 7-19, act as transitional passages between sections. The initial four pcs of each set, a form of 4-16, constitute a significant subcomponent of the larger set. George Perle has pointed out that these four pcs in measure 6, C-E-F-sharp-B, reiterate the opening four pcs of the first violin in measures 1 and 2.[18] The importance of these pcs, however, extends beyond this relationship. Pc set 4-16 is the only subset shared by 5-7 and 5-30, thus providing a link between pitch components of two separate sections. Furthermore, the two forms of 4-16 in measures 6 and 10 share the invariant pcs B and E, which are emphasized through their occurrence as sustained pedals throughout measures 7-9 in second violin and cello.

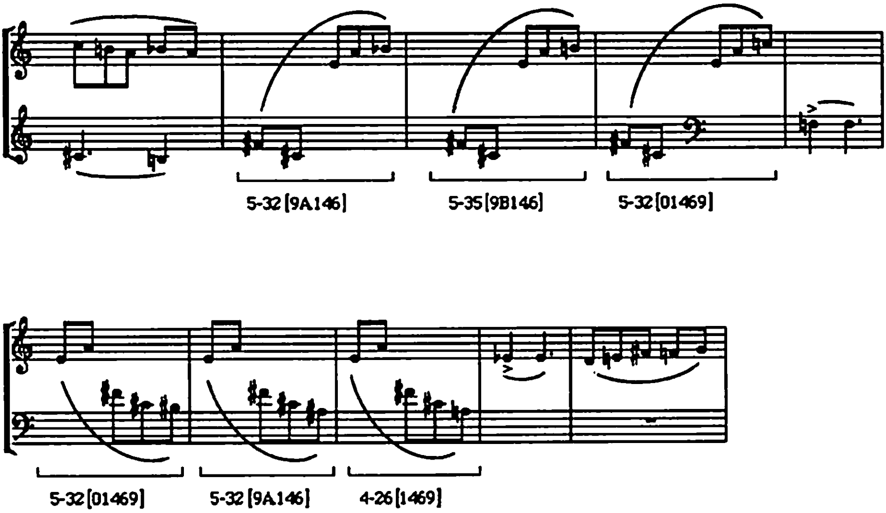
The inclusion of 5-7 of measure 4 in 6-5 of measures 1 and 2 provides an opportunity to demonstrate how the use of the SIA can help to better conceptualize this relationship. Figure 6 shows the SIAs of both sets and the grouping of the intervals in the SIA of 6-5 to derive the SIA of 5-7.

Figure 6.

We see from this that the omission of E-flat from 6-5 results in a form of 5-7. In musical terms, we can argue that Webern strongly emphasizes this inclusion relation through the difference in articulation and the registral separation between the cello E-flat and the tremolo dyads in the violins, which constitute a form of pc set 5-7.

The next passage I would like to examine is from Bartok's "Bulgarian Rhythm," contained in the *Mikrokosmos*, vol. IV, number 115. Figure 7 shows some of the pc sets in this passage.

Figure 7. Bartok, "Bulgarian Rhythm," *Mikrokosmos IV*, No. 115, mm. 8-17.



The two forms of 5-32 used here are inversionally related. Bartok makes this relation explicit in the melodic contours of measures 9 and 13. As figure 8 demonstrates, these forms of 5-32 have an inversional index of 10 with axes or centers of 5 and 11.

Figure 8.

9	A	1	4	6
<u>1</u>	<u>0</u>	<u>9</u>	<u>6</u>	<u>4</u>
A	A	A	A	A

Despite the fact that neither axis pc is present in the sets themselves, Bartok strongly emphasizes pc 5 (F) in m. 12, where the interruption of the previously consistent rhythm helps to create a cadential effect. The resultant agogic accent is one of several types of accent accorded this pitch, the others being the notated dynamic accent and the isolated registral placement. Another factor contributing to the significance of F at this point in the piece is its exclusion from the first 11 measures. Also of interest in this excerpt is how, in the second line of the example, Bartok immediately juxtaposes the two inversionally equivalent forms of 5-32 and then isolates their invariant subset, pc set 4-26.

My last excerpt is from Bartok's *Bagatelle*, op. 6, number 6. This passage, shown in figure 9, provides an interesting contrast between the diatonic and nondiatonic—a contrast that can be quite audible, even at a relatively elementary stage of instruction.

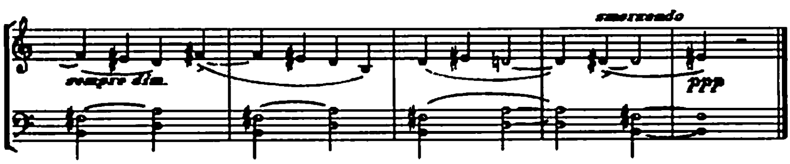
Figure 9. Bartok, Op. 6, No. 6, mm. 1-10.

The larger collections in measures 1-2 and 3-4 are forms of pc sets 7-Z38 and 8-18, respectively. By two of the usual measures of pc set similarity, neither one of these sets is closely related to the diatonic collection, pc set 7-35: 7-Z38 and 7-35 share only one common interval vector entry, while 8-18 contains no forms of 7-35. One of the striking aspects of these opening four measures is the way Bartok achieves a strong B orientation despite the relative scarcity of diatonic components. (Of course, we might perceive the notes C and E-sharp as modally influenced colorations of a basic diatonic collection.) The interplay between D-sharp and D-natural leaves unresolved the question of major or minor.

At measure 5 the music takes on a clearly diatonic profile: pc set 4-22 is a subset of the diatonic collection, 7-35. Moreover, 4-22 in this context embodies a strong triadic reference. Although pc set 3-11, the major or minor triad, is not the only trichordal subset of 4-22, it is certainly the most strongly emphasized due to its placement on the majority of the strong beats in measures 5 through 7.

Figure 10 shows the closing measures of the piece.

Figure 10. Bartok, Op. 6, No. 6, mm. 21-25.



With the entrance of E-sharp in the last measure, we might expect a resolution on F-sharp, which Bartok does not provide. One way to understand this cadence relates to the diatonic-nondiatonic polarity mentioned earlier: This cadence, identical to the form of pc set 4-Z29 appearing in measure 2 of figure 9, is a subset of the diatonic collection. Even though this set first appeared in m. 2 with the addition of D-natural at the end of the measure, it is only at the final cadence that its independence is asserted. In an immediate sense, the last E-sharp does not resolve. In a deeper sense, however, the cadence on 4-Z29, a subset of *both* the nondiatonic 7-Z38 and the diatonic 7-35, resolves our original diatonic-nondiatonic opposition.

In the three preceding analyses, I have attempted to show that set theory can be a powerful tool for describing the musical relationships within the passages. I must emphasize the word *musical* in my last statement, for in a theory or analysis class, numbers by themselves are meaningless unless they assist the student in developing and articulating analytical conclusions. Set theory, I believe, is not fundamentally incompatible with other modes of analytical reasoning. The essential element is an instructor who can select the necessary tools and present them to a class in a pedagogically effective and musically significant way. The goals of analysis must never be confused with its tools. This is as true for a beginning theory student as it is for the professional theorist.

NOTES

[1]Despite the title of the paper, my suggestions should prove useful in the graduate analysis class for the general music student without an extensive background in the analysis of 20th-century music.

[2]This, of course, does not apply to students who have some background in the sciences or mathematics.

[3]Ralph Turek, *The Elements of Music*, vol. 2 (New York: Alfred A. Knopf, Inc., 1988), 327-29.

[4]See, for example, Schoenberg's *Fundamentals of Music Composition* (London: Faber and Faber, 1967), chapter III ("The Motive") and "Analysis of the Four Orchestral Songs Opus 22" [*Perspectives of New Music* 3, no. 2 (Spring-Summer 1965): 1-21]. In his 1941 essay "Composition with Twelve Tones" [in *Style and Idea*, ed. Leonard Stein (Berkeley: University of California Press, 1975), 214-45], Schoenberg states that the basic set in a composition "functions in the manner of a motive" (219).

[5]Earl Henry, *Music Theory*, vol. II (Englewood Cliffs: Prentice-Hall, Inc., 1985), 340-47.

[6]The reader will note that I have just informally characterized the operation of a nexus set [see Allen Forte, *The Structure of Atonal Music* (New Haven and London: Yale University Press, 1973), 101].

[7]The analogy does not hold, of course, with seventh chords.

[8]Turek, *Elements*, 332; Bruce Benward, *Music in Theory and Practice*, 3rd ed., vol. 2 (Dubuque: Wm. C. Brown, 1977), 333.

[9]Turek, *Elements*, 332; Benward, *Theory and Practice*, 330-31. This contrasts with the traditional formulation of the set type or prime form as found in Forte, *Structure of Atonal Music*, 12. There the integers of a prime form represent not intervals, but pitch-classes derived through transposition of another form of the same set type. Allen Winold takes this approach in *Harmony: Patterns and Principles*, vol. II (Englewood Cliffs: Prentice-Hall, Inc., 1986), 252-53.

[10]Richard Chrisman, "Identification and Correlation of Pitch-Sets," *Journal of Music Theory* 15, nos. 1 and 2 (1971): 58-83 and "Describing Structural Aspects of Pitch-Sets Using Successive-Interval Arrays," *Journal of Music Theory* 21, no. 1 (Spring 1977): 1-28.

[11]In the Forte nomenclature, both sets of figure 1 are forms of 4-18: their inversional relationship will be discussed below.

[12]Forte, *Structure of Atonal Music*,8. Although Forte assumes 0 is C, the validity of the mathematical relationships, of course, do not depend on a fixed pitch-integer correspondence.

[13]See John Rahn, *Basic Atonal Theory* (New York and London: Longman, 1980), 49-51.

[14]In this and subsequent examples, A=pc 10 and B=pc 11. Note that the integers of each axis sum to the inversional index of example 3 or its mod 12 equivalent, 19.

[15]David Lewin, "Inversional Balance as an Organizing Force in Schoenberg's Music and Thought," *Perspectives of New Music* 6, no. 2 (Spring-Summer 1968): 1-21.

[16]The labeling of the pc sets in this and subsequent examples follows the conventions in Forte. With sets containing six or fewer pcs, the pcs included in the set are given in square brackets; with sets containing more than six pcs, the pcs excluded from the set are given in parentheses.

[17]Forte, *Structure of Atonal Music*, 49.

[18]George Perle, *Serial Composition and Atonality*, 5th ed. (Berkeley: University of California Press, 1981), 16.