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EXPLAINING INVERSION IN TWENTIETH-CENTURY THEORY

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A general intuitive meaning for the term inversion is not particularly difficult to explain or to understand. In music, inversion is the "substitution of higher for lower tones, and vice versa."[1] The student of tonal theory is usually acquainted with both harmonic and melodic inversion, and normally comfortable with the distinction between the two types. Often, however, the concept of inversion commonly used in twentieth-century theory—the inversion of a pitch-class set or a twelve-tone row—is difficult to understand intuitively for both the student and instructor. Pitch-class inversion is more closely related to the traditional notion of melodic inversion than harmonic inversion, and is often equated with strict or real melodic inversion. When the instructor or student of twentieth-century theory relates pitch-class inversion to strict melodic inversion, however, confusion and misunderstanding may arise, for there are important distinctions between them. A clearer understanding of the abstract process of pitch-class inversion used in twentieth-century theory may be reached first by examining the assumptions underlying the various types of inversion, second by exploring the different assumptions required to explain pitch-class inversion, and finally by utilizing and building on those assumptions to give an informal definition of pitch-class inversion. A brief survey of the literature concerning pitch-class inversion, examining several representative definitions of pitch-class inversion and methods for finding the inversion of a pitch-class set or twelve-tone row, concludes the discussion.

In a harmonic inversion the lowest member in an interval or chord moves to become the upper member or an upper voice in the interval or chord; a higher tone substitutes for a lower tone. The resulting interval or chord is said to be an inversion of the original interval or chord. Two assumptions underlie this definition of harmonic inversion: 1) the pitches are sounded simultaneously or are otherwise *temporally* unordered; and 2) the pitches also have a specific *registral* ordering, at least to some extent.

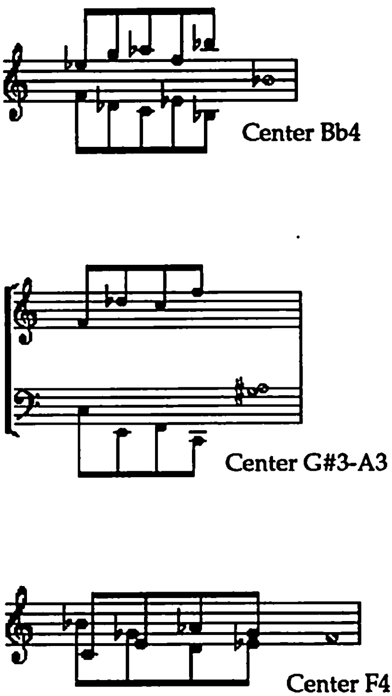
In a melodic inversion the direction of each interval in the melodic line is reversed; an ascending interval becomes a descending interval and vice versa. Again, a higher tone substitutes for a lower tone. Melodic inversion may be strict or tonal, depending on whether or not the exact size of the interval is preserved when the direction is changed. The resulting melodic line is said to be the inversion of the original melodic line. Two assumptions underlie this definition of melodic inversion: 1) the pitches occur in a specific *temporal* order; and 2) they are also *registrally* ordered, to the extent that each interval in the melody has a registral direction.

To fully understand the inversion of a pitch-class set, an unordered collection of pitch-classes without register, or to understand the inversion of a twelve-tone row, an ordered collection of all twelve pitch-classes without register, requires different assumptions. The assumptions made when describing a strict melodic inversion are not appropriate. Discarding these and replacing them with different assumptions allows an explanation for the process of inversion that does not rely on the specific register or specific order of the elements in the collection being inverted.

For the beginning student of twentieth-century theory, the most critical and frequently overlooked aspect of pitch-class inversion is the axis of symmetry. If two ordered or unordered collections of pitch-classes are related by inversion there must be an axis of symmetry. Understanding and explaining pitch-class inversion without reference to the axis of symmetry is difficult if not impossible. As stated earlier, pitch-class inversion is related to strict melodic inversion. The axis of symmetry in a pitch-class inversion is analogous to the center of inversion in a strict melodic inversion. The distinction in terminology between a center of inversion, for a strict melodic inversion, and an axis of symmetry, for a pitch-class inversion, is important as it provides a useful way to distinguish the concepts.

A strict melodic inversion has an inversional center, which is a single pitch, or a pair of pitches forming a half-step, in a specific register. The inversional center for a strict melodic inversion can be found by superimposing the two melodic lines that are related by inversion, and finding the pitch, or pair of pitches that lies exactly halfway between each corresponding pitch in the ordered melodic line. Figure 1 demonstrates the technique using several simple melodies and their inversions. This aspect of a strict melodic inversion is rarely considered. Since the relationship between the intervals in the inverted melodic line and the intervals in the original melodic line is clear, the definition of melodic inversion given above is sufficient, and thus the concept of inversional center is unnecessary. The concept, however, of an inversional center in a strict melodic inversion can be generalized to lead to the concept of an axis of symmetry in a pitch-class inversion, thereby providing a useful way to introduce that concept.

Figure 1.



An alternative definition of strict melodic inversion could be formulated if we consider that each pitch in a melodic line lies a certain interval above or below a specific inversional center. The strict inversion of a melody around the given center is the related melody whose pitches lie an equal distance from the center, but in the opposite direction. Once again, higher tones substitute for lower, and vice versa. The assumptions underlying this definition of melodic inversion are that the inversion has a center, and that the pitches have a specific registral position in relation to the center. Note that the exact order of the pitches in the melody is no longer a critical factor in this definition. In fact, a single pitch has a theoretical melodic inversion around a given center according to this definition.

A pitch-class inversion by assumption involves pitch-classes without specific register. Thus, the definition just presented must still be modified.

Rather than a center of inversion, a pitch-class inversion will have an axis of symmetry with no specific register, and consisting not of a single pitch-class or pair of pitch-classes, but represented by two pitch-classes or two pairs of pitch-classes separated by a tritone.

A convenient visual model for explaining pitch-class, axis of symmetry, and pitch-class inversion, is a circular diagram resembling a clock face, as shown in Figure 2.[2] The clock-face can be drawn with either letter names for pitch-classes, or integer names for pitch-classes. To understand how the entire spectrum of pitches in register can be represented in theory by a clock-face diagram, use the following steps. First, imagine a long string of wire, divided into equal parts and with pitch names, thus representing the entire spectrum of pitches in register. Next, imagine wrapping the string or wire into a spiral, matching points on the string with the same letter names, so, for example, C3, C4, and C5 are aligned. Finally, imagine looking at the spiral from the end, so it appears to be a circle. There is a position on the circle for each of the twelve chromatic pitches, yet no particular register can be isolated when the spiral is viewed from the end. It is important to remember that from one perspective the string or wire is still a spiral and contains marks representing the entire range of pitches in register, theoretically infinite in number. But by looking at the spiral from a different perspective, it is a closed circle, or considering the collection of all possible pitches without reference to specific register, there are only twelve distinct pitch-classes. Different aspects of the spiral are visible when viewed from different perspectives, just as different aspects of the fundamental musical materials are revealed when different assumptions are given.

An axis of symmetry can be represented on the clock-face diagram by using a single straight line that passes through the center of the clock, passing also through two antipodal "hours" representing pitch-classes on the clock, or through two antipodal "half-hours" representing pairs of pitch-classes on the clock. Figure 3 shows several different clock-face diagrams each with a specific axis of symmetry drawn. The process of pitch-class inversion around a specific axis of symmetry involves matching each pitch-class with its reflection across the line drawn to indicate the axis of symmetry. Figure 4 shows a clock-face diagram, with an axis of symmetry drawn, and with additional parallel lines drawn to indicate how the pitch-classes are matched for one specific axis of inversion. If the axis of symmetry is D-G-sharp, as shown, D-sharp will invert to C-sharp, E will invert to C, F will invert to B, and so forth. Or using pitch-class integer notation and the same axis of symmetry, the pitch-class set (1, 2, 3, 5, 7, 10) will be reflected into the pitch-class set (3, 2, 1, 11, 9, 6). Thus, the clock-face diagram provides an easy method of finding the pitch-class inversion for a pitch class set or twelve-tone row.

Figure 2.

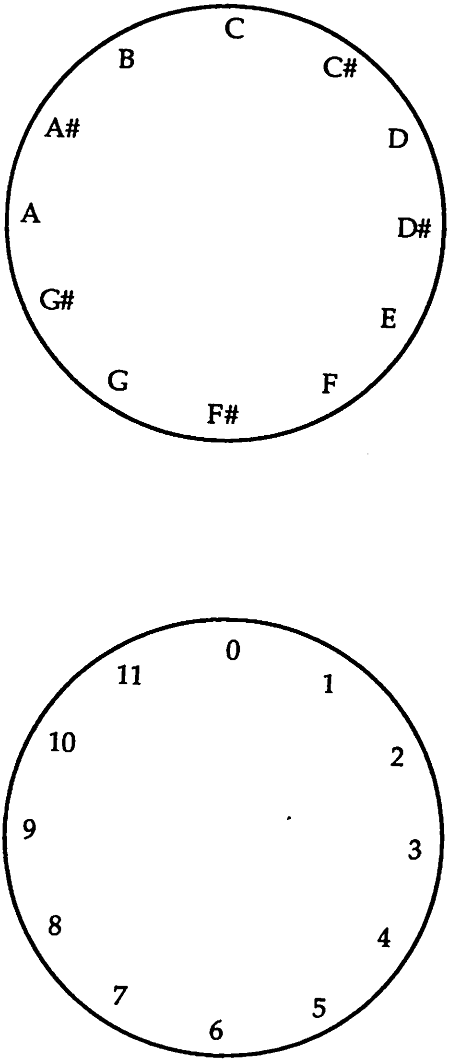


Figure 3.

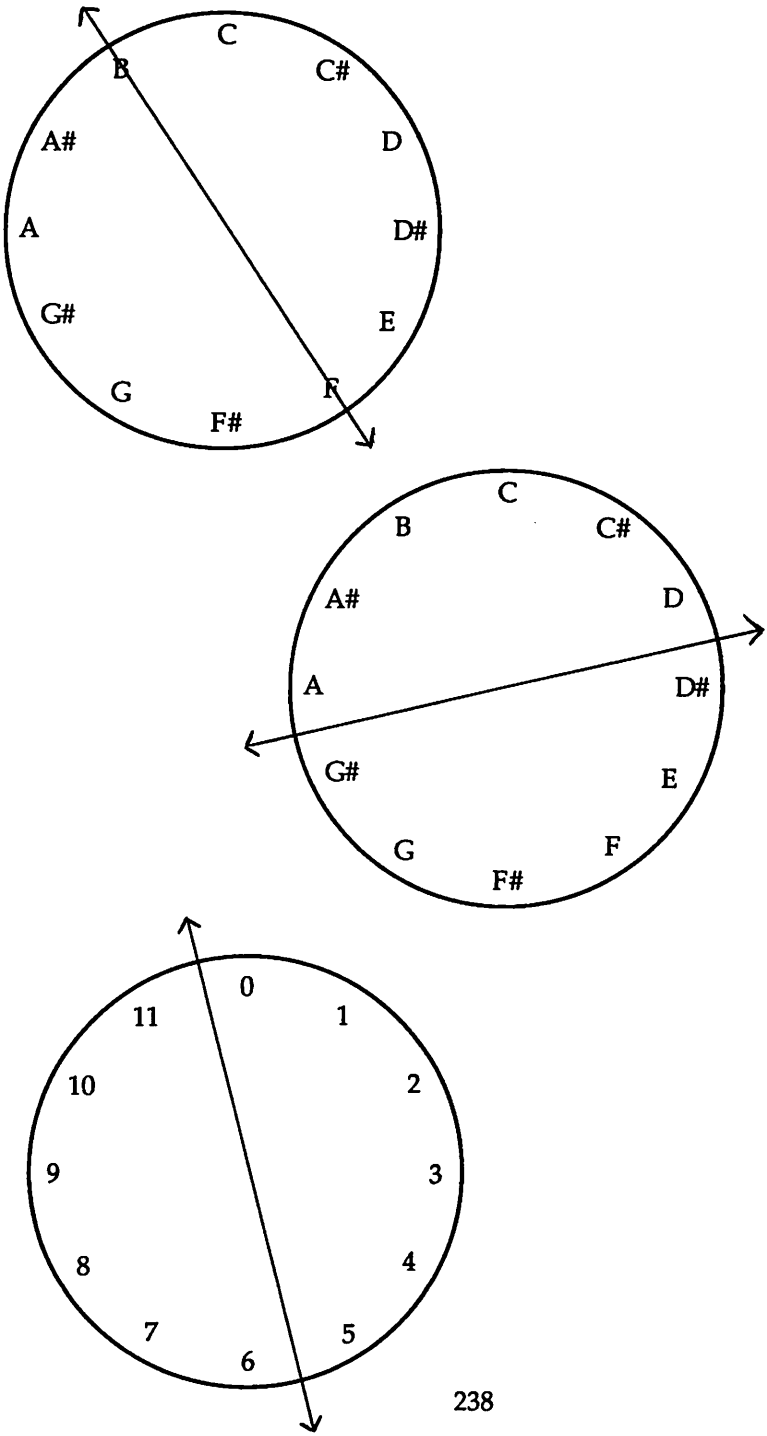


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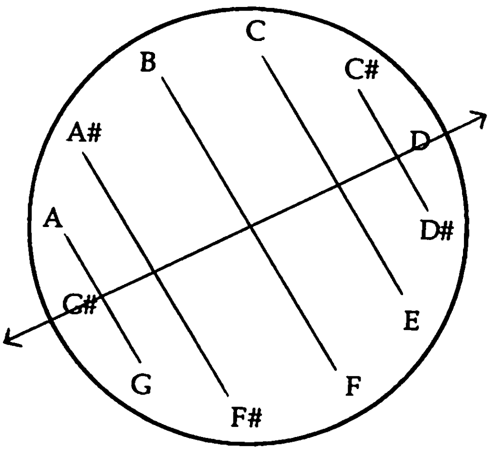
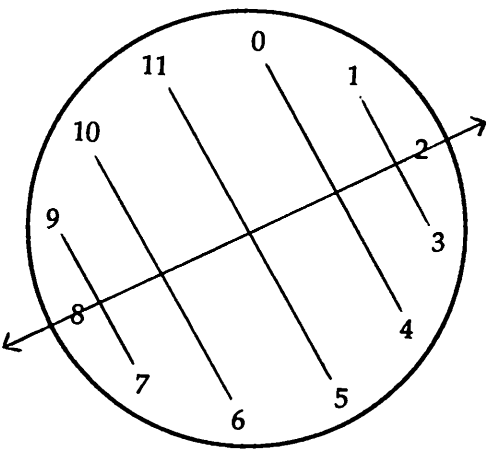
Figure 4.





The clearest way to define or describe the process of pitch-class inversion is to give the axis of symmetry. Since there is no specific register for each element in the set or order for the elements in the set, the assumptions for strict melodic inversion will not make sense. A new assumption, that the inversion must be around some specific axis of symmetry is required. Consider first the inversion of a single pitch-class. The inversion of a pitch-class "x" around a specific axis of symmetry is the pitch-class "y," which is the same distance from the axis of symmetry as the original pitch-class "x" but in the opposite direction, or on the opposite side of the clock-face. The inversion around a specific axis of symmetry of a pitch-class set, X, will be the pitch-class set, Y, whose members are the pitch-class inversions around a specific axis of symmetry of the members in the original set.

The literature contains many different definitions of pitch-class inversion, all of which depend on the explicitly or implicitly stated concept of an axis of symmetry. David Lewin defines pitch-class inversion informally when he writes "there are twelve ways of inverting the total chromatic into itself; one can fix any one of these inversions and regard the total chromatic as 'balanced' with respect to that inversion. Thus, e.g., the inversion that leaves D fixed also leaves A-flat fixed, and balances C-sharp with E-flat, C with E, B with F, etc.... We may think of these pitch-class inversions as possessing "axes"; each may be regarded, also as having a pair of antipodal 'centers.'"[3] Elsewhere, Lewin gives a formal definition of inversion: "We can ... define inversions independent of pc labeling. We shall do so by stipulating a pair (u, v) of pitch-classes such that the inversion is to reflect u into v, and we shall denote the inversion operation that does so by, 'I^{uv}.' Let us consider such an inversion: given a sample s, ... (where) s is n hours clockwise from u, the inverted image of s will appear reflected as n hours counterclockwise from v, the reflected image of u."[4] In each of Lewin's definitions the concept of an axis of symmetry is explicitly stated, and recognizing the assumption of a particular axis of symmetry for each inversion enables the reader to understand Lewin's definitions quickly.

George Perle gives a more concise definition for pitch-class inversion when he writes: "Where transpositionally related sets show the same difference for every pair of corresponding pitch classes, inversionally related sets show the same sum."[5] Perle represents pitch-classes utilizing integer notation, and performs addition and subtraction modulo 12. Perle's definition relies on an implicit assumption of an axis of symmetry. For when two pitch classes are related by inversion around a particular axis of symmetry, they will add modulo 12 to an integer that is twice the value of either of the two opposite poles of the axis of symmetry modulo 12. Note that if the axis of symmetry is made up of two pairs of pitch-classes the integers representing inversionally related pitch-classes will sum to an odd number, while if the axis of symmetry is made up of two single pitch-classes the integers representing inversionally related pitch-classes will sum to an even number. This definition and method for determining the inversional relationships among pitch-class sets and twelve-tone rows is very convenient, but without an understanding of the implicit axis of symmetry, may be confusing.

John Rahn, following Milton Babbitt and Allen Forte, considers inversion "as the compound operation *transposed inversion*."[6] His definition for pitch-class inversion, called T_nI, is easy to manipulate given pitch-class integer notation. He writes: "For any pc x and pc interval n, $T_nI(x)=-x+n$ (mod 12)."[7] The formula is easy to remember, and the modular arithmetic is easy to perform, but the notion of an axis of symmetry is not so easy to extract from this definition. The danger of considering inversion as a compound operation, fixing a single center, and transposing that inverted form to produce twelve other inverted forms is that one may forget there are indeed twelve different axes of inversion, independent but analogous to the twelve transpositional levels.[8] Later in his discussion of pitch-class inversion Rahn refers to the "two centers of symmetry 6 semitones apart."9 These centers of symmetry are "always one-half the index, n, of each operation T_nI . . . 1/2n can be either 1/2n or 1/2(n+12)=1/2n+6."[10] Rahn's "centers of symmetry" correspond to my axis of symmetry. The danger of considering inversion as a compound operation can be avoided if the student is frequently reminded to find the center of inversion using Rahn's formulas as well as the actual inverted form of a set or row.

These definitions of pitch-class inversion represent some of the many different methods for explaining pitch-class inversion found in the literature. Each method has strengths and weaknesses, each is appropriate in some analytic or theoretic situations but not all, and each is more easily understood if an intuitive meaning of pitch-class inversion is also presented. An intuitive understanding of a new concept is not always as practical or as easy to explain as a concrete formula that works to produce the correct answer. The student, however, who has both an intuitive grasp of a new concept, as well as a workable formula for practical applications, will certainly have the potential to gain a deeper understanding of that concept.

NOTES

[1]Willi Apel, "Inversion," 2nd edition., revised and enlarged (Cambridge: Belknap Press of Harvard University, 1977), p. 422.

[2]David Lewin, "Inversional Balance as an Organizing Force in Schoenberg's Music and Thought," *Perspectives of New Music* 6 (1968): 1-21.

[3]Ibid.

[4]David Lewin, "A Label-Free Development For 12-Pitch-Class Systems," *Journal of Music Theory* 21/1 (1977):35.

[5]George Perle, *Twelve-Tone Tonality* (Berkeley: University of California Press, 1977), p. 2.

[6]John Rahn, *Basic Atonal Theory* (New York: Longman Inc., 1980), p. 45. See also Milton Babbitt, "Twelve-Tone Invariants as Compositional Determinants," *The Musical Quarterly* 46 (1960): 252. Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973), pp. 7-11.

[7]Rahn, p. 47.

[8]David Lewin, "A Label-Free Development For 12-Pitch Class Systems," *Journal of Music Theory* 21/1 (1977) : 29-48.

[9]Rahn, p. 50.

[10]Ibid.